Servo Motor Performance Enhancement Using a Genetic Algorithm-Based Youla Kucera Controller

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ABSTRACT

In this paper a novel Youla-Kucera parameterization based robust controller is designed using multi-objective genetic algorithm. The design approach is developed for SISO system and is implemented on servo system. The design approach developed achieves multiobjectives related to robustness and disturbance rejection

KEYWORDS: Youla-Kucera parameterization , robust, disturbance rejection, servo system, multi-objective genetic algorithm

I. INTRODUCTION

In controller design, the Youla-Kucera Parameterization is useful as in it we can represent all the closed loop transfer functions in terms of Youla-Kucera parameter *Q*. This paper presents a methodology to address the problem of composite measure for multi-objective optimal performance of SISO systems. This technique solves H_∞ and time domain characteristic constraints problems of SISO systems, simultaneously. We get desired optimal multi-objective performance in the presence of conflicting objectives.

II. YOULA-KUCERA PARAMETERIZATION OF STABILIZING CONTROLLERS

Consider the plant of order n with $G_0(s)$ as nominal transfer function and the SISO feedback configuration as shown in the Figure 1[1].

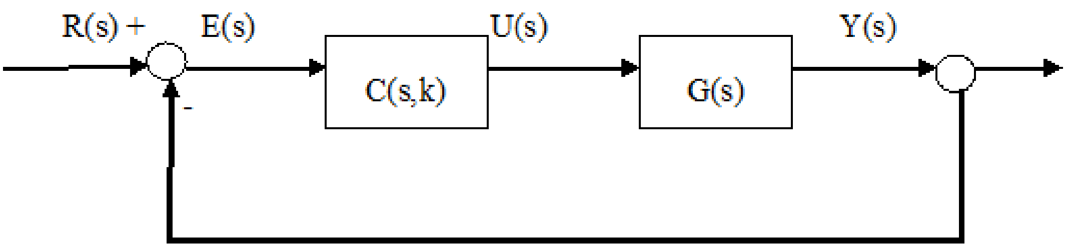

Figure 1: Control system consisting of a controller with fixed structure and a plant with model uncertainty

$$G_0(s) = \frac{b_n}{a_n} \tag{1}$$

Let a_n and b_n are coprime polynomials and $\hat{C}$ is the proper stabilizing controller of order m with coprime polynomials $\hat{x}_n$ and $\hat{y}_n$ in a feedback framework,

$$\hat{C}(s) = \frac{\hat{y}_n}{\hat{x}_n} \tag{2}$$

The system be internally bounded-input bounded-output (BIBO) stable which can be attained by utilizing the Youla-Kucera parameterization of stabilizing controllers [2-4].

It follows from the YK parameterization that stabilizing controllers are parameterized as given by

$$C(s) = \frac{y}{x} = \frac{\hat{y} - aQ}{\hat{x} - bQ} \tag{3}$$

$$\text{where, } a = \frac{a_n}{a_d}, b = \frac{b_n}{a_d}, \hat{x} = \frac{\hat{x}_n}{x_d}, \hat{y} = \frac{\hat{y}_n}{x_d}, Q = \frac{\hat{q}_n}{q_d} \tag{4}$$

a_d and x_d are two polynomials which form a polynomial $a_d x_d$ so that deg $a_d = n$ and deg $x_d = m$. Q is an arbitrary proper stable rational parameter, so that polynomial $\hat{q}_d$ is stable and polynomial $\hat{q}_n$ has same or lower degree [5].

Polynomial YK parameters can be recovered from the controller polynomials via the following relation:

$$\begin{bmatrix} q_n \\ q_d \end{bmatrix} = \begin{bmatrix} \hat{y}_n & -\hat{x}_n \\ a_n & b_n \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix} \tag{5}$$

YK parameters are polynomials.

III. OPTIMIZING CONTROLLER STRUCTURE WITH MULTI-OBJECTIVE OPTIMIZATION TECHNIQUE

Youla-Kucera can be used as structure of the controller which is then optimized with the multi-objective optimization technique. There is a trade-off between the controller order and the performance of the closed loop system. This multi-objective optimization problem gives a family of non-dominated or pareto optimal solutions. All the objective functions which the controller has to accomplished are contained in the evaluation function. The performance evaluation function used in this work has seven objectives. The first two functions are the condition for robust stability of the control system and the condition for disturbance rejection of the control system which are the H_∞ norm of the weighted complimentary sensitivity function and H_∞ norm of the sensitivity function in polynomial systems, respectively. The sensitivity of the control system from output *z* to the disturbance *d* is characterized by closed loop sensitivity function *S*, [4], given by

$$S = \frac{1}{1 + \frac{b_n y_n}{a_n x_n}} = \frac{a_n x_n}{a_n x_n + b_n y_n} \tag{6}$$

H_∞ norm of the weighted sensitivity function *S* is:

$$\|S\|_\infty = \left\| W_d \frac{a_n x_n}{a_n x_n + b_n y_n} \right\|_\infty \tag{7}$$

and complementary sensitivity function is *T*, is given by

$$T = 1 - S = 1 - \frac{a_n x_n}{a_n x_n + b_n y_n} = \frac{b_n y_n}{a_n x_n + b_n y_n} \tag{8}$$

H_∞ norm of the weighted complimentary sensitivity function *T* is given by

$$\|T\|_\infty = \left\| W_m \frac{b_n y_n}{a_n x_n + b_n y_n} \right\|_\infty \tag{9}$$

In order to assure good time response performance, five-time domain objectives included are rise time, settling time, peak, overshoot, undershoot. The control problem is to design a controller, C, so that the resulting feedback system should have no undershoot, minimal undershoot, minimal settling time, minimal rise time, optimum peak and rejects the disturbance effectively.

IV. EXAMPLE

To illustrate the method, a detailed design example is presented. Consider the control system shown in the Figure 2. The model of the plant, servomotor taken from [6] is represented by the following transfer function

$$G_0(s) = \frac{20}{s(s+1)} \tag{10}$$

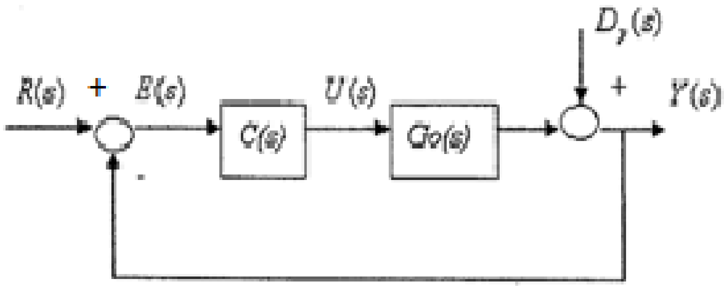

Figure 2: Control system with uncertainty and disturbance acting on the plant output

$G_o(s)$ is the nominal plant and $C(s,k)$ is the Youla-Kucera controller.
Let $b_1 = 1$,
Youla-Kucera parameter is taken as

$$q = \frac{a_1 s + a_0}{b_1 s^2 + b_2 s + b_a} \tag{11}$$

The vector k of youla-kucera parameter is given by $k = [a_0, a_1, b_1, b_2]^T$ which is to be obtained solving the multi-objective optimization problem.

The Plant model, using multiplicative uncertainty is given by

$$G(s) = G_0(s)[1 + \Delta(s) W_m(s)] \tag{12}$$

where, $G_0(s)$ is the nominal transfer function of the plant, the plant perturbation $\Delta(s)$ is assumed to be stable but uncertain, where the weighting function $W_m(s)$ is stable and known.

The multiplicative uncertainty $W_m(s)$ for robustness is taken as [7];

$$W_m(s) = \frac{0.1}{s^2 + 0.1s + 10} \tag{13}$$

First objective function $f1$ for robust stability in multi-objective optimization is

$$f1 = \left\| W_m \frac{b_n y_n}{a_n x_n + b_n y_n} \right\|_\infty \tag{14}$$

The weighting function $W_d(s)$ for disturbance rejection is taken as [7];

$$W_d(s) = \frac{1}{(s+1)} \tag{15}$$

The error signal $E(s)$, assuming the input signal to be a unit step, is evaluated as follows:

$$E(s) = \frac{1}{1 + C(s,k) G_0(s)} R(s) \tag{16}$$

The second objective function f2 for the disturbance rejection in multi-objective optimization is

$$f2 = \left\| W_d \frac{a_n x_n}{a_n x_n + b_n y_n} \right\|_\infty \tag{17}$$

The H_∞ norm is calculated using MATLAB function normhinf.
The controller parameter vector was searched within the following bounds:

$$a0 = [-20]; a1 = [-40,0]; b2 = [0,30]; b3 = [0,500]$$

By solving the optimization problem using MOGA, the following pareto front is obtained

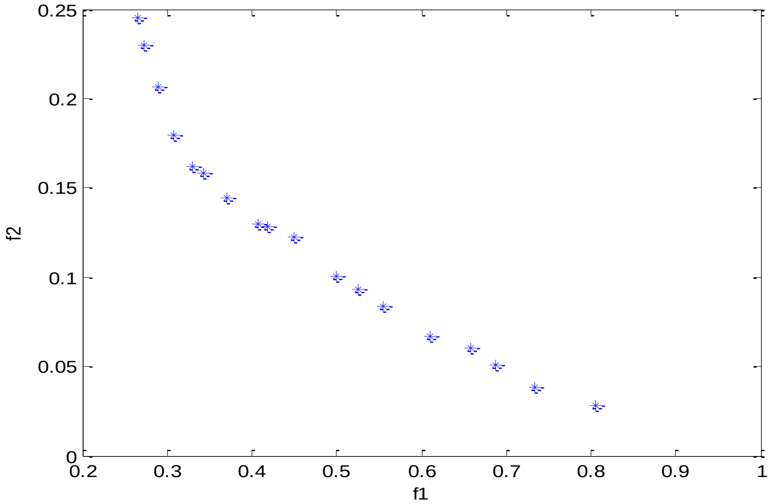

Figure 3: Pareto front between f1and f2

Pareto front obtained in Figure 3 shows that when f1 increases f2 decreases.

The comparison between multi-objective Youla-Kucera controllers using MOGA optimization technique and controller developed in [6] is shown in Figure 4.

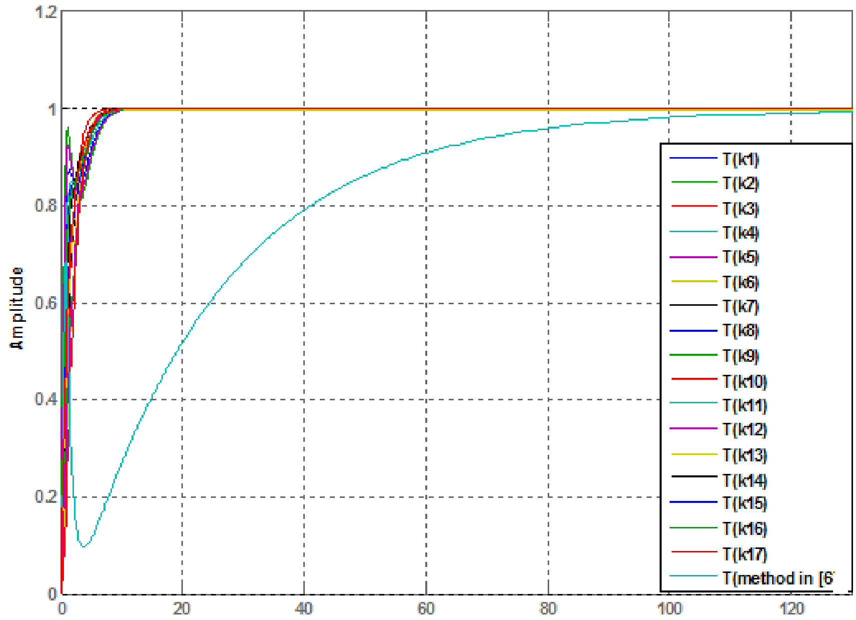

Figure 4: Step response of YK controllers designed with MOGA and method developed in [6]

With optimal solution vector k1, the step responses obtained are shown in the Figure 5 and Figure 6

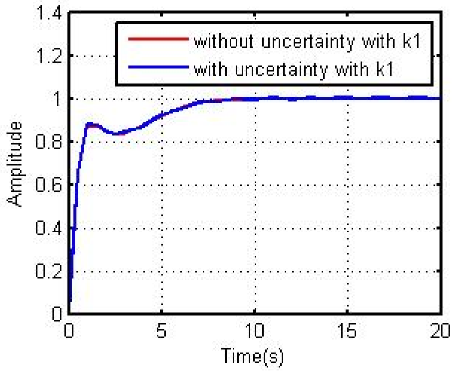

Figure 5: Step response of the plant using k1 with and without uncertainty

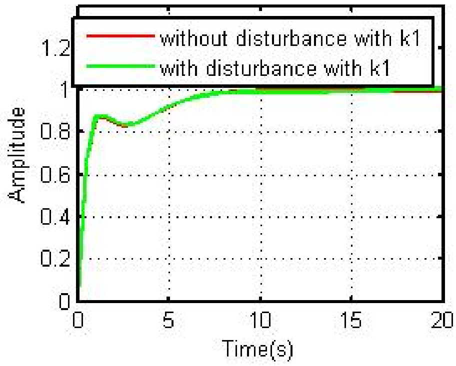

Figure 6: Step response of the controlled plant using k1 with and without disturbance

With optimal solution vector k2, the step responses obtained are shown in the Figure 7 and Figure 8.

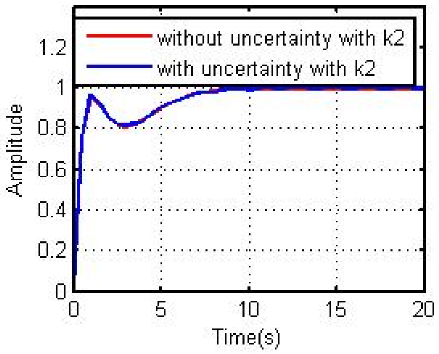

Figure 7: Step response of the plant using k2 with and without uncertainty

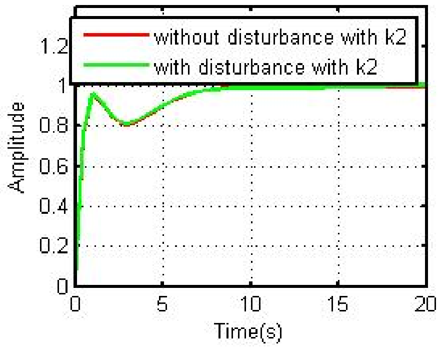

Figure 8: Step response of the controlled plant using k2 with and without disturbance

With optimal solution vector k8, the step responses obtained are shown in the Figure 9 and Figure 10.

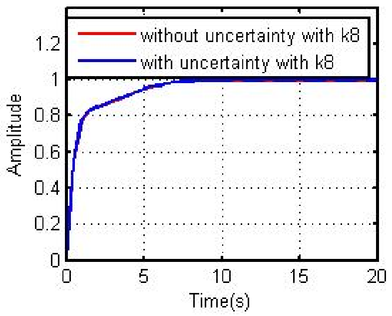

Figure 9: Step response of the plant using k8 with and without uncertainty

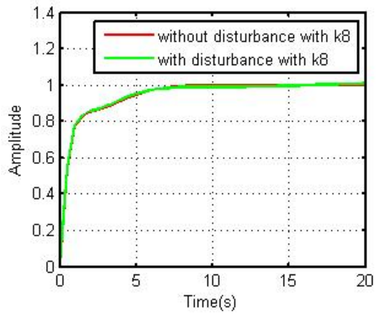

Figure 10: Step response of the controlled plant using k8 with and without disturbance

With optimal solution vector k9, the step responses obtained are shown in the Figure 11 and Figure 12.

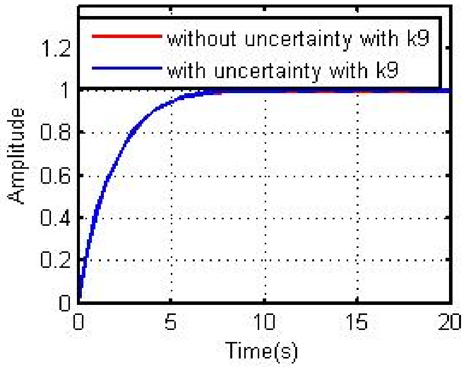

***Figure 11:* Step response of the plant using k9 with and without uncertainty**

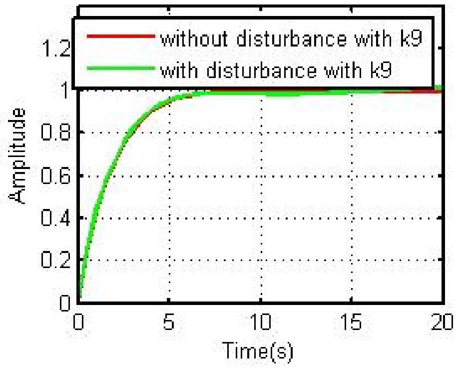

***Figure 12:* Step response of the controlled plant using k9 with and without disturbance**

With optimal solution vector k16, the step responses obtained are shown in the Figure 13 and Figure 14.

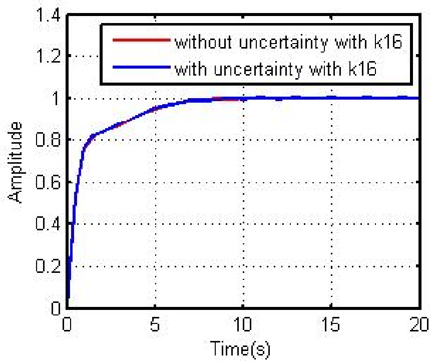

Figure 13: Step response of the plant using k16 with and without uncertainty

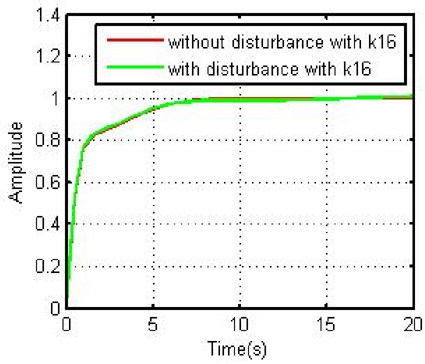

Figure 14: Step response of the controlled plant using k16 with and without disturbance

With optimal solution vector k17, the step responses obtained are shown in the Figure 15 and Figure 16.

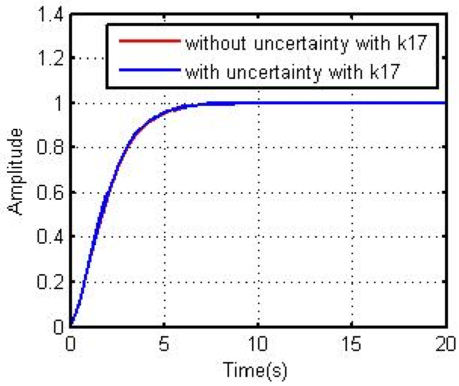

***Figure 15:* Step response of the plant using k17 with and without uncertainty**

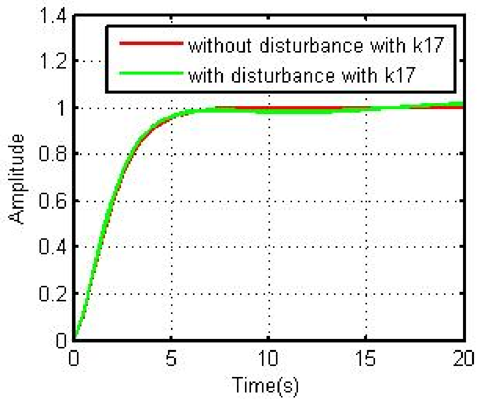

***Figure 16:* Step response of the controlled plant using k17 with and without disturbance**

The designed multi-objective controllers based on Youla-Kucera parameterization using MOGA optimization technique have better tracking performance as compared to controller designed in [6] as observed in Figure 4. The Table 1 provides the time domain performances with different YK controller parameters and controller designed in [6]. Overshoot in all the Youla-Kucera controllers is zero.

Table 1: Time domain performances achieved by different Youla-Kucera controllers

S.No	Controller Parameters	Rise Time(s)	Settling Time(s)	Peak(s)
1	-13.9681, -35.1619,5.8825,305.3379	4.4414	7.2547	1.0000
2	-14.4226, -42.0459,5.9426,305.5810	0.5504	7.6107	0.9990
3	-9.8776, -9.2915,5.5629,303.1235	2.7911	4.7727	0.9990
4	-14.0715, -30.1832,5.8691,303.7410	3.9126	6.8781	0.9998
5	-14.3014, -39.2763,5.8784,305.3946	0.7922	7.4739	0.9993
6	-3.4217, -3.9720,5.4292,300.2193	3.3734	5.6587	0.9990
7	-9.3547, -13.1023,5.7594,302.0050	3.1709	5.6501	0.9990
8	-14.1123, -27.1486,5.9934,304.1224	3.5074	6.5842	0.9997
9	-3.6698, -9.5104,5.3277,301.5148	3.8471	6.3190	0.9999
10	-10.8642, -16.1105,5.7847,302.1719	3.1130	5.7627	1.0000
11	-13.4556, -23.9739,5.7596,303.8973	3.2825	6.3316	0.9998
12	-3.3018, -7.4861,5.3626,300.7535	3.7298	6.1491	0.9987
13	-9.8989, -14.9117,5.6683,303.3478	3.2212	5.8077	0.9983
14	-13.3353, -21.3108,5.7562,303.7084	2.9907	5.9920	0.9998
15	-10.2586, -20.2272,5.4330,302.0545	3.6628	6.4415	0.9987
16	-13.6111, -26.4131,5.7753,303.1691	3.5565	6.5847	0.9999
17	0, -1.1554,5.1909,300.1652	3.4815	5.9794	0.9998
18	Controller designed in [6]	57.3726	95.9679	0.9991

The designed multi-objective controllers based on Youla-Kucera parameterization using MOGA optimization technique have lower rise time, setting time and peak as compared to controller designed in [6] as observed in Table 1. The effectiveness of this novel design approach is tested on a servo system which provides excellent tracking in the presence of uncertainties and disturbances

V. REFERENCES

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