Bipolar Vague S-Spaces: Analyzing Structure and Interactions

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ABSTRACT

In this work, we introduce the notion of S-space and almost S-space on bipolar vague topological space with some of its properties. The interrelations between newly defined spaces and other spaces are also discussed.

KEYWORDS: Bipolar vague G_δ -set, Bipolar vague F_σ -set, Bipolar vague S-space, Bipolar vague submaximal space, Bipolar vague almost S-spaces

I. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by Zadeh [12]. The theory of fuzzy topological space was studied and developed by C.L.Chang [3]. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Later, the vague set theory was introduced by W. L. Gau and D. J. Buehrer [5], as a generalizations of Zadeh's fuzzy set theory.Bipolar-valued fuzzy sets, which are introduced by Lee [8], are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. Using the notion of bipolar-valued fuzzy sets, Jun et al. [6] dealt with subalgebras and closed ideals of BCH-algebras based on bipolar-valued fuzzy sets. A.K. Mishra [9] introduced the concepts of P-spaces as generalizations of ω_α -additive spaces of Sikorski [10] and C. Goffman [4]. The concept of p-spaces in fuzzy setting was defined by G. Balasubramanian [11]. Amost P-spaces in classical topology was initiated by R. Levy [7].

In this paper several characterizations of bipolar vague S-space and bipolar vague almost S-space are studied and the conditions under which an bipolar vague submaximal space becomes a bipolar vague S-space are also analyzed.

II. PRELIMINARIES

Definition 2.1[8]: Let X be the universe. Then a bipolar valued fuzzy set, A on X is defined by positive membership function μ_A^+, that is $\mu_A^+ : X \to [0,1]$, and a negative membership function μ_A^-, that is $\mu_A^- : X \to [-1,0]$. For the sake of simplicity, we shall use the symbol $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

Definition 2.2[5]: A vague set A in the universe of discourse U is a pair (t_A, f_A) where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are the mapping such that $t_A + f_A \leq 1$ for all $u \in U$ .The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A, 1 - f_A]$ is called the vague value of u in A, and denoted by $v_A(u)$, i.e $v_A(u) = [t_A(u), 1 - f(u)]$.

Definition 2.3[1]: Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form $A = \{\langle x, [t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)]\rangle : x \in X\}$ where $[t_A^+, 1 - f_A^+] : X \to [0,1]$ and $[-1 - f_A^-, t_A^-] : X \to [-1,0]$ are the mapping such that $t_A^+ + f_A^+ \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $v_A^+ = [t_A^+, 1 - f_A^+]$ and $v_A^- = [-1 - f_A^-, t_A^-]$.

Definition 2.4[1]: A bipolar vague set $A = [v_A^+, v_A^-]$ of a set U with $v_A^+ = 0$ implies that $t_A^+ = 0, 1 - f_A^+ = 0$ and $v_A^- = 0$ implies that $t_A^- = 0, -1 - f_A^- = 0$ for all $x \in U$ is called zero bipolar vague set and it is denoted by 0.

Definition 2.5[1]: A bipolar vague set $A=[v_A^+, v_A^-]$ of a set U with $v_A^+=1$ implies that $t_A^+=1, 1-f_A^+=1$ and $v_A^-=-1$ implies that $t_A^-=-1, -1-f_A^-=-1$ for all $x \in U$ is called unit bipolar vague set and it is denoted by 1.

Definition 2.6[1]: Let $A=<x,[t_A^+, 1-f_A^+][-1-f_A^-, t_A^-]>$ and $B=<x,[t_B^+, 1-f_B^+][-1-f_B^-, t_B^-]>$ be two bipolar vague sets then their union, intersection and complement are defined as follows:

1. $A \cup B=\{(x,[t_{A\cup B}^+(x), 1-f_{A\cup B}^+(x)], [-1-f_{A\cup B}^-(x), t_{A\cup B}^-(x)]) / x \in X\}$ where
   $t_{A\cup B}^+(x)=\max\{t_A^+(x), t_B^+(x)\}$, $t_{A\cup B}^-(x)=\min\{t_A^-(x), t_B^-(x)\}$ and
   $1-f_{A\cup B}^+(x)=\max\{1-f_A^+(x), 1-f_B^+(x)\}$,
   $-1-f_{A\cup B}^-(x)=\min\{-1-f_A^-(x), -1-f_B^-(x)\}$.
2. $A \cap B=\{(x,[t_{A\cap B}^+(x), 1-f_{A\cap B}^+(x)], [-1-f_{A\cap B}^-(x), t_{A\cap B}^-(x)]) / x \in X\}$ where
   $t_{A\cap B}^+(x)=\min\{t_A^+(x), t_B^+(x)\}$, $t_{A\cap B}^-(x)=\max\{t_A^-(x), t_B^-(x)\}$ and
   $1-f_{A\cap B}^+(x)=\min\{1-f_A^+(x), 1-f_B^+(x)\}$,
   $-1-f_{A\cap B}^-(x)=\max\{-1-f_A^-(x), -1-f_B^-(x)\}$.
3. $\overline{A}=\{(x,[f_A^+(x), 1-t_A^+(x)], [-1-t_A^-(x), f_A^-(x)]) / x \in X\}$ for all $x \in X$.

Definition 2.7[1]: Let (X, BV_τ) be a bipolar vague topological space and $A=\langle x,[t_A^+, 1-f_A^+][-1-f_A^-, t_A^-]\rangle$ be a BVS in X. Then the bipolar vague interior and bipolar vague closure of A are defined by,

$$Bvcl(A)=\cap\{K : K \text{ is a BVCS in } X \text{ and } A \subseteq K\},$$
$$Bv\operatorname{int}(A)=\cup\{G : G \text{ is a BVOS in } X \text{ and } G \subseteq A\}$$

Note that $bvcl(A)$ is a BVCS and $bv\operatorname{int}(A)$ is a BVOS in X. Further,

1. A is a BVCS in X iff $Bvcl(A)=A$,
2. A is a BVOS in X iff $Bv\operatorname{int}(A)=A$

Definition 2.8[2]: A bipolar vague set A in a bipolar vague topological space (X, τ) is called bipolar vague dense if there exists no bipolar vague closed B in (X, τ) such that $A \subset B \subset 1$.

Definition 2.9[2]: A bipolar vague set A in a bipolar vague topological space (X, τ) is called bipolar vague nowhere dense set if there exists no bipolar vague open set B in (X, τ) such that $B \subset BVcl(A)$. That is, $BV\operatorname{int}(BVcl(A))=0$.

Definition 2.10[2]: Let (X, τ) be a bipolar vague topological space. A vague set A in (X, τ) is called bipolar vague first category if $A=\bigcup_{i=1}^{\infty} A_i$, where A_i's are bipolar vague nowhere dense sets in (X, τ).

The complement of a bipolar vague first category sets in (X, τ) is a bipolar vague residual set in (X, τ).

Definition 2.11[2]: Let (X, τ) be bipolar vague topological space. Then (X, τ) is said to bipolar vague Baire space if $BV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i)=0$, where A_i's are bipolar vague nowhere dense sets in (X, τ).

Definition 2.12[2]: A bipolar vague topological space (X,τ) is called a bipolar vague D-Baire space if every bipolar vague first category set in (X,τ) is a bipolar vague nowhere dense set in (X,τ). That is, (X,τ) is a bipolar vague D-Baire space if $BV\operatorname{int}(BVcl(A))=0$ for each bipolar vague first category set A in (X,τ).

III. BIPOLAR VAGUE S-SPACE

Definition 3.1: A bipolar vague set A in a bipolar vague topological space (X,τ) is called a bipolar vague $G_\delta-$set in (X,τ) if $A=\bigcap_{i=1}^{\infty}A_i$ where $A_i\in\tau$ for $i\in I$.

Definition 3.2: A bipolar vague set A in a bipolar vague topological space (X,τ) is called a bipolar vague $F_\sigma-$set in (X,τ) if $A=\bigcup_{i=1}^{\infty}A_i$ where $\overline{A_i}\in\tau$ for $i\in I$. In other words, the complement of bipolar vague $G_\delta-$set in (X,τ) is a bipolar vague $F_\sigma-$set in (X,τ).

Definition 3.3: A bipolar vague topological space (X,τ) is called a bipolar vague S-space if countable intersection of bipolar vague open sets in (X,τ) is bipolar vague open. i.e. every non-zero bipolar vague $G_\delta-$set in (X,τ) is bipolar vague open in (X,τ).

Theorem 3.4: If A is a non zero bipolar vague $F_\sigma-$set in a S-space (X,τ), then A is a bipolar vague closed set in (X,τ).

Proof: Let A be a non-zero bipolar vague $F_\sigma-$set in (X,τ), $A=\bigcup_{i=1}^{\infty}A_i$ where $A_i's$ are bipolar vague closed in (X,τ). Then $\overline{A}=\overline{\bigcup_{i=1}^{\infty}A_i}=\bigcap_{i=1}^{\infty}\overline{A_i}$. Now $A_i's$ are bipolar vague closed in (X,τ), implies that $\overline{A_i}'s$ are bipolar vague open in (X,τ). Hence we have $\overline{A}=\bigcap_{i=1}^{\infty}\overline{A_i}$, where $\overline{A}\in\tau$. Then $\overline{A}$ is a bipolar vague $G_\delta-$set in (X,τ). Since (X,τ) is a bipolar vague S-space, $\overline{A}$ is a bipolar vague open in (X,τ). Therefore A is a bipolar vague closed set in (X,τ).

Theorem 3.5: If the bipolar vague topological space (X,τ) is a bipolar vague S-space, then $BV\operatorname{int}(\bigcap_{i=1}^{\infty}A_i)=\bigcap_{i=1}^{\infty}BV\operatorname{int}(A_i)$ where $A_i's$ are bipolar vague open sets in (X,τ).

Proof: Let $A_i's$ be non-zero bipolar vague open sets in a bipolar vague S-space (X,τ). Then $A=\bigcap_{i=1}^{\infty}A_i$ is a bipolar vague G_δ -set in (X,τ). Since (X,τ) is a bipolar vague S-space, the bipolar vague G_δ -set A is bipolar vague open in (X,τ). Hence, we have $BV\operatorname{int}(A)=\operatorname{int}(A)$. This implies that $BV\operatorname{int}(\bigcap_{i=1}^{\infty}A_i)=\bigcap_{i=1}^{\infty}A_i=\bigcap_{i=1}^{\infty}BV\operatorname{int}(A_i)$ and hence $BV\operatorname{int}(\bigcap_{i=1}^{\infty}A_i)=\bigcap_{i=1}^{\infty}BV\operatorname{int}(A_i)$ where $A_i's$ are bipolar vague open sets in (X,τ).

Theorem 3.6: If A_i's are bipolar vague closed sets in a bipolar vague S-space (X,τ), then $BVcl(\bigcup_{i=1}^{\infty}A_i)=\bigcup_{i=1}^{\infty}A_i$.

Proof: Let A_i 's be a bipolar vague closed sets in a bipolar vague S-space (X,τ) which implies that $\overline{A_i}$'s are bipolar vague open sets in (X,τ). Let $\mu = \bigcap_{i=1}^{\infty} \overline{A_i}$. Then μ is a non-zero bipolar vague $G_\delta -$set in (X,τ). Since the bipolar vague topological space (X,τ) is a bipolar vague S-space, $BV\operatorname{int}(\mu) = \mu$, which implies that $BV\operatorname{int}(\bigcap_{i=1}^{\infty} \overline{A_i}) = \bigcap_{i=1}^{\infty} \overline{A_i}$. Then $\overline{BVcl(\bigcup_{i=1}^{\infty} A_i)} = \overline{\bigcup_{i=1}^{\infty} A_i}$. Hence we have $BVcl(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} A_i$.

Theorem 3.7: If the bipolar vague topological space (X,τ) is a bipolar vague S-space and if A is a bipolar vague first category set in (X,τ), then A is not a bipolar vague dense set in (X,τ).

Proof: Assume the contrary, suppose that A is a bipolar vague first category set in (X,τ) such that $BVcl(A) = 1$. Then, $A = \bigcup_{i=1}^{\infty} A_i$, where A_i 's are bipolar vague nowhere dense sets in (X,τ). Now $\overline{BVcl(A_i)}$ is a bipolar vague open set in (X,τ). Let $\mu = \bigcap_{i=1}^{\infty} \overline{BVcl(A_i)}$. Then μ is a non-zero bipolar vague $G_\delta -$set in (X,τ). Now we have $\bigcap_{i=1}^{\infty} \overline{BVcl(A_i)} = \overline{\bigcup_{i=1}^{\infty} BV\operatorname{int}(A_i)} \subseteq \overline{\bigcup_{i=1}^{\infty} A_i} = \overline{A}$. Hence $\mu \subseteq \overline{A}$. Then $BV\operatorname{int}(\mu) \subseteq BV\operatorname{int}(\overline{A}) = \overline{BVcl(A)} = 0$ ie., $BV\operatorname{int}(\mu) = 0$. Since (X,τ) is a bipolar vague S-space, $\mu = BV\operatorname{int}(\mu)$ which implies that $\mu = 0$, a contradiction to μ being a non-zero bipolar vague $G_\delta -$set in (X,τ). Hence $BVcl(A) \neq 1$.

Definition 3.8: A bipolar vague topological space (X,τ) is called a bipolar vague submaximal space if for each bipolar vague set A in (X,τ) such that $BVcl(A) = 1$, then $A \in \tau$ in (X,τ).

Theorem 3.9: If each bipolar vague $G_\delta -$set is a bipolar vague dense set in a bipolar vague submaximal space (X,τ), then (X,τ) is a bipolar vague S-space.

Proof: Let A be a bipolar vague $G_\delta -$set in a bipolar vague submaximal space (X,τ). Then, by hypothesis, A is a bipolar vague dense set in (X,τ). Since (X,τ), is a fuzzy submaximal space, the bipolar vague dense set A in (X,τ) is a bipolar vague open set in (X,τ). i.e., every bipolar vague $G_\delta -$set in (X,τ) is a bipolar vague open set in (X,τ). Therefore (X,τ) is a bipolar S-space.

Theorem 3.10: If $BV\operatorname{int}(A) = 0$, where A is a bipolar vague $F_\sigma -$set in a bipolar vague submaximal space (X,τ), then (X,τ) is a bipolar vague S-space.

Proof: Let A be a bipolar vague $G_\delta -$set in a bipolar vague submaximal space (X,τ). Then, $\overline{A}$ is a bipolar vague $F_\sigma -$set in (X,τ). Then, by hypothesis, $BV\operatorname{int}(\overline{A}) = 0$ for a vague $F_\sigma -$set A in (X,τ). This implies that $BVcl(A) = 1$. Then A is a bipolar vague dense set in (X,τ). since (X,τ) is a bipolar vague submaximal space, the bipolar vague dense set A in (X,τ) is a bipolar vague open set in (X,τ). i.e., every bipolar $G_\delta -$set in (X,τ) is a bipolar vague open set in (X,τ). Therefore (X,τ) is a bipolar vague S-space.

Theorem 3.11: If each bipolar vague $F_\sigma -$set is a bipolar vague nowhere dense set in a bipolar vague submaximal space (X,τ), then (X,τ) is a bipolar vague S-space.

Proof: Let A be a bipolar vague $F_\sigma -$set in a bipolar vague submaximal space (X,τ) such that $BV\operatorname{int}(BVcl(A)) = 0$. Then, $BV\operatorname{int}(A) \subseteq BV\operatorname{int}(BVcl(A))$, implies that $BV\operatorname{int}(A) = 0$. Now

$BV\text{int}(A)=0$ for a bipolar vague $F_\sigma -$set in a bipolar vague submaximal space (X,τ). Then, by Theorem 3.10, (X,τ) is a bipolar vague S-space.

Theorem 3.12: If $BVcl(BV\text{int}(A))=1$, for each bipolar vague $G_\delta -$set A in a bipolar vague submaximal space (X,τ). Then (X,τ) is a bipolar vague S-space.

Proof: Let A be a bipolar vague $F_\sigma -$set in a bipolar vague submaximal space (X,τ). Then $\overline{A}$ is a bipolar vague $G_\delta -$set in (X,τ). By hypothesis, $BVcl(BV\text{int}(\overline{A}))=1$. Then $\overline{BVcl(BV\text{int}(\overline{A})}=0$. This implies that $(\overline{\overline{BV\text{int}(BVcl(A))}})=0$. i.e. $BV\text{int}(BVcl(A))=0$ and hence A is a bipolar vague nowhere dense set in (X,τ). Thus the bipolar vague $F_\sigma -$set A is a bipolar vague nowhere dense set in a bipolar vague submaximal space (X,τ). Hence by Theorem 3.11, (X,τ) is a bipolar vague S-space.

Theorem 3.13: If A is a bipolar vague residual set in a bipolar vague submaximal space (X,τ), then A is a bipolar vague $G_\delta -$set in (X,τ).

Proof: Let A be a bipolar vague residual set in a bipolar vague submaximal space (X,τ) . Then $\overline{A}$ is a bipolar vague first category set in (X,τ) and hence $\overline{A}=\bigcup_{i=1}^{\infty}A_i$, where A_i are bipolar vague nowhere dense sets in (X,τ). Since $A_i's$ are bipolar vague nowhere dense set $BV\text{int}(BVcl(A_i))=0$. Then, $BV\text{int}(A_i)\subseteq BV\text{int}(BVcl(A_i))$ implies that $BV\text{int}(A_i)=0$. This implies that $\overline{BV\text{int}(A_i)}=1$ and hence $BVcl(\overline{A_i})=1$. Since (X,τ) is a bipolar vague submaximal space, the bipolar vague dense sets in $\overline{A_i}'s$ are bipolar vague open sets in (X,τ). Then $A_i\,'s$ are bipolar vague closed sets in (X,τ). Hence $\overline{A}=\bigcup_{i=1}^{\infty}A_i$ where $A_i's$ are bipolar vague closed sets in (X,τ) implies that $\overline{A}$ is a bipolar vague $F_\sigma -$set in (X,τ). Therefore A is a bipolar vague $G_\delta -$set in (X,τ).

Theorem 3.14: If A is a bipolar vague residual set in a bipolar vague submaximal and bipolar vague S-space (X,τ), then A is a bipolar vague open set in (X,τ).

Proof: Let A be a bipolar vague residual set in a bipolar vague submaximal and bipolar vague S-space (X,τ). Since A is bipolar vague residual set in a bipolar vague submaximal space (X,τ), by Theorem 3.13, A is a bipolar vague $G_\delta -$set in (X,τ). Since (X,τ) is a bipolar vague S-space, the bipolar vague $G_\delta -$set in (X,τ) is a bipolar vague open set in (X,τ) . Hence a bipolar vague residual set A in a bipolar vague submaximal and bipolar vague S-space (X,τ) is a bipolar vague open set in (X,τ).

Theorem 3.15: If A is a bipolar vague nowhere dense set in a bipolar vague submaximal space (X,τ), then A is a bipolar vague closed set in (X,τ) .

Proof: Let A be a bipolar vague nowhere dense set in a bipolar vague submaximal space (X,τ). Then we have $BV\text{int}(BVcl(A))=0$ and $BV\text{int}(A)\subseteq BV\text{int}(BVcl(A))$, implies that $BV\text{int}(A)=0$. Then $\overline{BV\text{int}(A)}=1$ implies that $BVcl(\overline{A})=1$ and hence $\overline{A}$ is a bipolar vague dense set in (X,τ). Since (X,τ) is a bipolar vague submaximal space, $\overline{A}$ is a bipolar vague open set in (X,τ). Therefore the bipolar vague nowhere dense set A is a bipolar vague closed in (X,τ).

Theorem 3.16: If a bipolar vague S- space (X,τ) is a bipolar vague submaximal and bipolar vague Baire space, then (X,τ) is a bipolar vague D-Baire space.

Proof: Let the bipolar vague S-space (X,τ) be a bipolar vague submaximal and bipolar vague Baire space. Let A be a bipolar vague first category set in (X,τ). Since (X,τ) is a bipolar vague Baire space, $BV\text{int}(A)=0$. Then, $\overline{BV\text{int}(A)}=1$. This implies that $BVcl(\overline{A})=1$ and hence $\overline{A}$ is a bipolar vague dense set in (X,τ). Since (X,τ) is a bipolar vague submaximal space, $\overline{A}$ is a bipolar vague open set in (X,τ). Then, A is a bipolar vague closed set in (X,τ) and hence $BVcl(A)=A$. Now $BV\text{int}(BVcl(A))=BV\text{int}(A)$, implies that $BV\text{int}(BVcl(A))=0$. Then A is a bipolar vague nowhere dense set in (X,τ). Hence, each bipolar vague first category set in (X,τ) is a bipolar vague nowhere dense set in (X,τ). Therefore (X,τ) is a bipolar vague D-Baire space.

Definition 3.17: A bipolar vague topological space (X,τ) is called a *bipolar vague almost S –space* if for every non-zero bipolar vague G_δ – set in (X,τ), $BV\text{int}(A)\neq 0$ in (X,τ).

Definition 3.18: Let $A=<x,[t_A^+,1-f_A^+][-1-f_A^-,t_A^-]>$ in a bipolar vague topological space (X,τ) is said to be bipolar vague regular closed set if $A=BVcl(BV\text{int}(A))$

Theorem 3.19: If A is a bipolar vague F_σ – set in a bipolar vague almost S-space (X,τ), then, $BVcl(A)\neq 1$.

Proof: Let A be a bipolar vague F_σ – set in a bipolar vague almost S-space (X,τ) .Then, $\overline{A}$ is a bipolar vague G_δ – set in (X,τ) .Since (X,τ) is a bipolar vague almost S-space, for the bipolar vague G_δ – set $\overline{A}$,we have $BV\text{int}(\overline{A})\neq 0$. This implies that $\overline{BVcl(A)}\neq 0$ and hence we have $BVcl(A)\neq 1$.

Theorem 3.20: If each non-zero bipolar vague G_δ – set is a bipolar vague regular closed set in a bipolar vague topological space (X,τ), then (X,τ) is a bipolar vague almost S-space.

Proof: Let A be a non-zero bipolar vague G_δ – set in (X,τ) such that $BVcl(BV\text{int}(A))=A$. We claim that $BV\text{int}(A)\neq 0$. Assume the contrary. Then $BV\text{int}(A)=0$ will imply that $BVcl(BV\text{int}(A))=BVcl(0)=0$ and hence we will have $A=0$, a contradiction to A being a non-zero bipolar vague G_δ – set in (X,τ). Hence we must have $BV\text{int}(A)\neq 0$, for a bipolar vague G_δ – set in (X,τ) and therefore (X,τ) is a bipolar vague almost S-space.

Theorem 3.21: If each non-zero bipolar vague first category set is a bipolar vague dense set in a bipolar vague topological space (X,τ), then (X,τ) is not a bipolar vague almost S-space.

Proof: Let A be a bipolar vague first category set in (X,τ) such that $BVcl(A)=1$. Then $A=\bigcup_{i=1}^{\infty}A_i$, where A_i 's are bipolar vague nowhere dense sets in (X,τ). Now $\overline{BVcl(A_i)}$ is a bipolar vague open set in (X,τ). Let $\mu=\bigcap_{i=1}^{\infty}\overline{BVcl(A_i)}$ then μ is a G_δ – set in (X,τ). Now $\mu=\bigcap_{i=1}^{\infty}\overline{BVcl(A_i)}=\overline{\bigcup_{i=1}^{\infty}BV\text{int}(A_i)}\subseteq\overline{\bigcup_{i=1}^{\infty}A_i}=\overline{A}$, i.e., $\mu\subseteq\overline{A}$. Then $BV\text{int}(\mu)\subseteq BV\text{int}(\overline{A})$ and hence $BV\text{int}(\mu)\subseteq\overline{BVcl(A)}=0$, i.e., $BV\text{int}(\mu)=0$. Hence, for the bipolar vague G_δ – set μ in (X,τ), $BV\text{int}(\mu)=0$. Therefore (X,τ) is not a bipolar vague almost S-space.

Theorem 3.22: If δ is a bipolar vague residual set in a bipolar vague almost S-space (X,τ), then there exists a bipolar vague G_δ – set μ in (X,τ) such that $\mu\subseteq\delta$ in (X,τ).

Proof: Let δ be a bipolar vague residual set in (X,τ). Then, $\overline{\delta}$ is a bipolar vague first category set in (X,τ). Let $\lambda=\overline{\delta}$. Then $\lambda=\bigcup_{i=1}^{\infty}\lambda_i$, where λ_i 's are bipolar vague nowhere dense sets in (X,τ). Now $\overline{BVcl(\lambda_i)}$ is a bipolar vague open set in (X,τ). Let $\mu=\bigcap_{i=1}^{\infty}\overline{BVcl(\lambda_i)}$. Then μ is a $G_\delta-$set in (X,τ). Since (X,τ) is a bipolar vague almost S-space, $BV\operatorname{int}(\mu)\neq 0$, in (X,τ). Let $BV\operatorname{int}(\mu)=\gamma$, where $\gamma\in\tau$. Now $\mu=\bigcap_{i=1}^{\infty}\overline{BVcl(\lambda_i)}=\overline{\bigcup_{i=1}^{\infty}BV\operatorname{int}(\lambda_i)}\subseteq\overline{\bigcup_{i=1}^{\infty}\lambda_i}=\overline{\lambda}$ i.e., $\mu\subseteq\delta$. Thus, if δ is a bipolar vague residual set in bipolar vague almost S-space (X,τ), then there exists a bipolar vague $G_\delta-$set μ in (X,τ) such that $\mu\subseteq\delta$ in (X,τ).

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