Exploring Maxwell's Equations and Lorentz Transformation in Free and Curved Space

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ABSTRACT

Using the expression of Lorentz force of the electron a useful expression for Lorentz transformation of the special relativity theory was found. The Lorentz transformation in a curved space which account for the effect of fields is also found. This relationship resembles that of generalized special relativity anfreduces to that of Einstein special relativity in the absence of fields.

Keywords: *Lorentz transformation, Lorentz force, fields, curved space.*

I. INTRODUCTION

Einstein special relativity (SR) is one of the big achievements that change the classical concept of absolute space and time coordinate.The theory of relativity resulted from an analysis of the physical consequences implied by the absence of a universal frame of reference. The special theory of relativity developed by Albert Einstein in 1905, treats problems involving inertial frames of reference, which are frames of reference moving at constant velocity with respect to one another. The special theory of relativity is based upon two postulates. The first states that the laws of physics take the same form in all inertial frames. The second postulate states that the speed of light is constant and has the same value in all inertial frames. Special relativity succeeded in describing a wide variety of physical phenomena that concerns high speed particles in free space. However it does not account for the effect of fields on space, time and mass. Despite the fact that general relativity accounts for the effect of the gravitational field, the theory of relativity does not explain the effect of other fields on space, time and mass. For instance the crystal field is found to change the mass of electrons and photons. The neutrinos masses are also found to be affected during their journey from the sun to the earth .This necissates searching for promoting SR to be sensitive to the fields effects[1,2,3].

II. DISPLACEMENT CURRENT LORENTZ TRANSFORMATIONS

Consider the magnetic field generated by displacement current

$$\nabla \times H = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \quad (1)$$

But

$$\underline{B} = \mu \underline{H}$$

$$\frac{1}{\mu} \nabla \times B = \varepsilon \frac{\partial E}{\partial t} \quad (2)$$

$$\nabla \times B = \mu\varepsilon \frac{\partial E}{\partial t} \quad (3)$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \mu\varepsilon \frac{\partial E}{\partial t}$$

$$[\frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x]\hat{k} = \mu\varepsilon \frac{\partial E_z}{\partial t} \hat{k}$$

Consider the solution

$$B_y = B_0 e^{i(k.r - \omega t)} \quad (4)$$

$$E_z - E_0 e^{i(k.r-\omega t)} (5)$$
$$\frac{\partial B_y}{\partial x} = ikB_y (6)$$
$$\frac{\partial E_z}{\partial t} - i\omega E_z (7)$$
$$\mu\varepsilon = \frac{1}{c^2}$$
$$ikB_y = -i\mu\varepsilon\omega E_z$$
$$B_y = -\mu\varepsilon \frac{\omega}{k} E_z = -\frac{1}{c^2}\left(\frac{2\pi f\lambda}{2\pi}\right)$$
$$= -\frac{1}{c^2}(f\lambda)E_z = -\frac{c}{c^2}E_z$$
$$B_y = -\frac{E_z}{c}$$
$$B_y^{'} = -\frac{E_z^{'}}{c} (8)$$

Sub these relations in (4) and (5) given

$$E_z^{'} = \gamma\left(E_z - \frac{v}{c}E_z\right) = \gamma\left(1 - \frac{v}{c}\right)E_z$$
$$E_z = \gamma\left(E_z^{'} + \frac{v}{c}E_z^{'}\right) = \gamma\left(1 + \frac{v}{c}\right)E_z^{'} (9)$$

The termγ can be found by using relations (10) and (9) to get

$$E_z = \gamma^2\left(1 - \frac{v}{c}\right) = \gamma\left(1 + \frac{v}{c}\right)E_z (10)$$

Hence

$$\gamma^{-2} = \left(1 - \frac{v^2}{c^2}\right)$$
$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} (11)$$

Which is the ordinary SR transformation coefficient when the space is permeated with field the space is deformed [ ]. Thus

$$c^2 d\tau^2 = c^2 g_{00}(2)dt^2 - g_{xx}(2)dx^2$$
$$= c^2 g_{00}(1)dt^2 - g_{xx}(1)dx^2$$

Thus

$$c^2\big(g_{00}(2) - g_{00}(1)\big)dt^2 = [g_{xx}(2) - g_{xx}(1)]dx^2$$
$$[c^2 dg_{00}]dt^2 = [dg_{xx}]dx^2 (12)$$

But

$$g_{xx} = -g_{00}^{-1}$$

Thus

$$dg_{xx} = g_{00}^{-2} dg_{00} (13)$$

Inserting (13) in (12) gives

$$c^2 g_{00}^2 dt^2 = dx^2 (14)$$

Hence the velocity is gives by

$$v = \frac{dx}{dt} = cg_{00} = c\left(1 + \frac{2\phi}{c^2}\right) (15)$$

Consider a particle having initial velocity v_0 initial potentialϕ_0, final velocity v and final potential ϕ_1.According to equation (15), one gets

$$v_0 = c\left(1 + \frac{2\phi_0}{c^2}\right)$$
$$v = c\left(1 + \frac{2\phi_1}{c^2}\right) (16)$$

Hence

$$v - v_0 = \frac{2}{c}[\phi_1 - \phi_0] = \frac{2\phi}{c}$$

$$v = v_0 + \frac{2\phi}{c} \quad (17)$$

Thus the average mean velocity is given by

$$v_m = \frac{v + v_0}{2} = \frac{v + v - \frac{2\phi}{c}}{2}$$

$$v_m = v - \frac{\phi}{c} \quad (18)$$

Assuming that $v$stands for the mean velocity $v_m$in equation (11), one gets

$$\gamma = \left(1 - \frac{{v_m}^2}{c^2}\right)^{-\frac{1}{2}} \quad (19)$$

Inserting equation (18) in (19) gives

$$\gamma = \left[1 - \frac{\left(v - \frac{\phi}{c}\right)^2}{c^2}\right]^{-\frac{1}{2}} \quad (20)$$

It is very interesting to note that when no field exist equation (20) becomes that of SR

III. DISCUSSION

The SR Lorentz transformation can be found by using the electromagnetic force relation for a charged electron moving in an electromagnetic field, as shown by equations (2), (3), (5.2.4), and (5).

Using Maxwell equations, concerning generation of electric field by variable magnetic field, one gets a relation between Ycomponent of the magnetic field and Z component of electric field in equation (12). Using all above relations the Einstein SR coefficient γ is shown to be typical to that of SR.Coefficient γ for particles moving in a field is found by using ordinary relation between velocity, acceleration and potential per unit mass [see equations (1 to 8). fortunately this relation reduces to that of SR in the absence of afield, as equation (9) indicate replacing the velocity v with the average velocity v_m in SR Einstein coefficient in (10), one gets γ in terms of v_m. Again using the relations between velocity and potential per unit mass, one gets two different expressions for γ depending on the time and time free relation of v and v_0[see equations (19), (26)] . fortunately the two expressions reduces to that of SR, as shown by equations (20) and (27).According to equations (5) the electromagnetic wave is a travelling wave in the x-direction, with a magnetic field vibrating in the y-direction and the electric field is vibrating in the Z-direction. According to this version by and E_z are related according to equation (7). Using the electromagnetic force relations (5.2.4) and (5), one gets relations in equations (9) and (10) which relates E_z to E_z'. These two relations are used to derive Einstein coefficientγ, which is strikingly the same as that of SR.To find γ for any field, one uses the invariance of the interval to get an expression which relates the initial velocity v_0 to the final velocity v[see equation (17)]. This equation is the Einstein counter part of the Newton one which is given by

$$v = v_0 - at(22)$$

This equationwhich reflects space curvature can be written as

$$v = v_0 - \frac{2\phi}{c} \quad (23)$$

But

$$g_{00} = \left(1 + \frac{2\phi}{c}\right)$$

Thus the velocity time evolution is described by time metric as

$$v = v_0 + g_{00} - 1(24)$$

By replacing c by

$$c_m = \frac{c^2}{2}$$

And replacing c_m by c again one gets from (14)

$$dx = c\left(1+\frac{\phi_1}{c^2}\right)dt = \left(c+\frac{\phi_1}{c}\right)dt(26)$$

One can find the same relation by adopting an approximation which assumes that

$$g_{xx} = 1 \qquad g_{00} = \left(1+\frac{2\phi}{c^2}\right)(25)$$

By assuming $d\tau$to be very small equation of interval gives

$$dx = g_{00}^{\frac{1}{2}}dt$$

But

$$g_{00}^{\frac{1}{2}} = \left(1+\frac{2\phi}{c^2}\right)^{\frac{1}{2}} = 1+\frac{\phi}{c^2}$$

Thus

$$v = \frac{dx}{dt} = \left(1+\frac{\phi}{c^2}\right)c$$

When

$$v = v_0\phi = \phi_0(27)$$

Thus

$$v_0 = c+\frac{\phi_0}{c}(28)$$

Hence

$$v = v_0+\left(\frac{\phi_1-\phi_0}{c}\right) = v_0+\frac{\phi}{c}(29)$$

But

$$\phi = ax(30)$$

Thus (29) becomes

$$v = v_0+\frac{ax}{c}(31)$$

For a photon

$$x = ct(32)$$

Thus

$$v = v_0+at(33)$$

Which is the ordinary Newton second Low.

By replacing v by the mean velocity v_m in (11) then using equations (18) and (19), one gets γ for any field ϕ.To incorporate the effect of the field one can also use a relation between field potential per unit mass and the time metric g_{00} to derive anew Lorentz transformation coefficientγ. According to the expression for interval (22) a useful relation for $vandL$ in terms of ϕ were found in equations (32) and (33). These were used to express x in terms of x' and vice versa [see (14) and (15), then a field dependent relation for γ are found in (20) .This expression fortunately reduced to that of SR in the absence of field.

IV. CONCLUSION

The expression of the electric and magnetic force on the electrons beside the expression of the displacemet current is used to derive special relativistic and generalized special relativistic Lorentz transformation ,using the concept of curved space. This can successfully describe a wide variety of physical phenomena in the precence and absence of fields.

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