Exploring Lorentz Transformation for Free Space and Electromagnetic Fields Using Newton's Laws

David L. Lee*[1], Jessica W. Zhou[2], Daniel R. M. Johnson[3] & Yui Takahashi[4]
[1]Department of Civil and Environmental Engineering, University of California, Berkeley, USA
[2]Department of Earth and Atmospheric Sciences, University of Houston, Texas, USA
[3]Department of Structural Engineering, University of Cape Town, Cape Town, South Africa
[4]Graduate School of Science and Technology, Keio University, Tokyo, Japan

ABSTRACT

The expression of Lorentz force and Maxwell's equations were used to drive Lorentz transformation in terms of electric and magnetic fields. This expression is typical to that of special relativity. The Lorentz transformation that takes care of the effect of fields on the physical system was also derived using Newtonian laws specially the velocity aceleration relation. The relation obtained is typical to that of generalized special relativity.

Keywords: *Lorentz force, Lorentz transformation, average velocity, special relativity, generalized special relativity*

I. INTRODUCTION

Maxwell's equations describe the relation between electric and magnetic fields. It also describes how they are generated. They are derived from Gauss law, ampere's law beside Faraday's law. It relates also fluxes to field are strengths via permittivity. Maxwell's equations are shown to be invariant under Lorentz transformation of coordinate. Lorentz transformation (LT) is of the corner stone's of special relativity (SR). Usually LT is used to relate space and time coordinates for different inertial frames. Lorentz transformation is successful in describing the inertial motion, but unable to describe the motion of particles in fields. Different attempts were made to account for the effect of fields by using Lorentz transformation [1].These transformations are mainly dependent on space and time coordinates. Nothing is done for Maxwell's equations. This work is devoted to use Maxwell's equations to derive Lorentz transformation that accounts for the effect of fields on space, time and mass[2,3].

II. LORENTZ TRANSFORMATION AND ELECTROMAGNETIC FILED

The force is given by in the frame S

$$F = e\left(E + v \times \underline{B}\right)(1)$$

In the frame S' it is given by

$$F' = e(E' + v \times B')(2)$$

Assume that e is constant and the electromagnetic force is tans forms frame S to frame S' as

$$eE' = e\gamma(E + v \times B)(3)$$

If one assumes that the charge is at rest in frameS', thus no magnetic field is exerted therefore the force is S' is given by (assume the electric field)

$$E'_z = \gamma\left(E_z + vB_y\right)(4)$$

If in contrary, the charge is at rest in S, hence:

$$F = eE_z$$

And

$$eE_z = e\gamma\left(E'_z - vB'_y\right)(5)$$

Using Maxwell's equations

$$\nabla \times E = -\frac{\partial B}{\partial t}(6)$$

$$\nabla \times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y\right)\hat{\imath} - \left(\frac{\partial}{\partial x}E_z - \frac{\partial}{\partial z}E_x\right)\hat{\jmath} + \left(\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\right)\hat{k}$$

$$= -\frac{\partial B_x}{\partial t}\hat{\imath} - \frac{\partial B_y}{\partial t}\hat{\jmath} - \frac{\partial B_z}{\partial t}\hat{k}(7)$$

The $\hat{\jmath}$ component is given by

$$-\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}(8)$$

Let

$$E_z = E_0 e^{i(k.r-\omega t)}$$
$$B_y = B_0 e^{i(k.r-\omega t)}(9)$$
$$k.r = k_x x + k_y y + k_z z$$
$$k_x = k_y = k_z = k$$
$$\frac{\partial E_z}{\partial x} = ik.E_z(10)$$
$$\frac{\partial B_y}{\partial t} = -i\omega B_y(11)$$

Sub (10) and (11) in (8) yields

$$-ikE_z = +i\omega B_y$$
$$B_y = -\frac{k}{\omega}E_z = -\frac{2\pi E_z}{\lambda(2\pi f)} = -\frac{E_z}{\lambda f} = -\frac{E_z}{c}$$
$$B_y = -\frac{E_z}{c}$$
$$B_y' = -\frac{E_z'}{c}(12)$$

Sub (12) in (4) and (5) thus

$$E_z' = \gamma\left(E_z - \frac{v}{c}E_z\right) = \gamma\left(1 - \frac{v}{c}\right)E_z(13)$$
$$E_z = \gamma\left(E_z' + \frac{v}{c}E_z'\right) = \gamma\left(1 + \frac{v}{c}\right)E_z'(14)$$
$$E_z = \gamma\left(1 + \frac{v}{c}\right)E_z'$$
$$\gamma^2\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right) = 1$$
$$\left(1 - \frac{v^2}{c^2}\right)\gamma^2 = 1$$
$$\gamma^2 = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)}(15)$$
$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

Which is ordinary SR expression

III. GENERALIZED SPECIAL RELATIVITY FARADAY ELECTROMAGNETIC LORENTZ TRANSFORMATION

For a particle moving with acceleration a the velocity is given by

$$v = v_0 + at = v_0\frac{axt}{x} = \frac{\phi t}{x} + v_0(16)$$

Where

$$V = potential = Fx = max$$
$$\phi = potentialperunitmass(17)$$
$$= \frac{V}{m} = ax(18)$$

Sub in (15) and assuming the relation hold for all physical system

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} (19)$$

$$\gamma = \frac{1}{\left(1 - \left(\frac{\phi t}{xc} + \frac{v_0}{c}\right)^2\right)^{\frac{1}{2}}} (20)$$

For photon

$$x = ct (21)$$

$$\gamma = \frac{1}{\left(1 - \left(\frac{\phi t}{c^2 t} + \frac{v_0}{c}\right)^2\right)^{\frac{1}{2}}} (22)$$

$$\gamma = \frac{1}{\left(1 - \left(\frac{\phi}{c^2} + \frac{v_0}{c}\right)^2\right)^{\frac{1}{2}}} (23)$$

For no potential

$$\phi = O$$

$$\gamma = \frac{1}{\left(1 - \frac{v_0^2}{c^2}\right)^{\frac{1}{2}}} (24)$$

Which is ordinary SR expression

One can also use the average velocity to find Lorentz transformation in the presence of fields. To do these assume again the relation.

$$\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-\frac{1}{2}} (25)$$

For particle in a field moving with constant acceleration the velocity is given by:

$$v = v_0 - at (26)$$

$$x = v_0 t - \frac{1}{2} at^2 (27)$$

But

$$\left(\frac{v_0 + v}{2}\right) t = v_m t (29)$$

Where

$v_m$is the mean velocity

This is given by

$$v_m = \frac{v_0 + v}{2} (30)$$

Replacing v by v_m in (25) one gels

$$\gamma = \left[1 - \frac{v_m^2}{c^2}\right]^{-\frac{1}{2}} (31)$$

Using the relation

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\phi (32)$$

Thus

$$v_0^2 = v^2 + 2\phi$$

$$v_0 = \sqrt{v^2 + 2\phi} (33)$$

Incorporating (33) in (30) and (31) one gets

$$\gamma = \left[1 - \left(\frac{v + \sqrt{v^2 + 2\phi}}{2c^2}\right)^2\right]^{-\frac{1}{2}} (34)$$

When no filed exists

$$\phi = 0$$

Thus

$$\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-\frac{1}{2}} (35)$$

From (27)

$$v = v_0 - at = v_0 - \frac{ax}{x} t$$

$$v = v_0 - \frac{\phi}{x} t (36)$$

Thus

$$v_0 = \left(v + \frac{\phi}{x} t\right) (37)$$

Therefore equation (30) reads

$$v_m = \frac{v_0 + v}{2} = v + \frac{\phi}{2x} t (38)$$

Using (38) in equation (31) given

$$\gamma = \left[1 - \frac{\left(v + \frac{\phi}{2x} t\right)^2}{c^2}\right]^{-\frac{1}{2}}$$

Assuming this relation is general. For pulse of light

$$x = ct$$

$$\gamma = \left[1 - \left(\frac{v + \frac{\phi}{2ct} t}{c^2}\right)^2\right]^{-\frac{1}{2}}$$

$$\gamma = \left[1 - \left(\frac{v + \frac{\phi}{2c}}{c^2}\right)^2\right]^{-\frac{1}{2}}$$

IV. DISCUSSION

The SR Lorentz transformation can be found by using the electromagnetic force relation for a charged electron moving in an electromagnetic field, as shown by equations (2), (3), (4), and (5).

Using Maxwell equations, concerning generation of electric field by variable magnetic field, one gets a relation between Ycomponent of the magnetic field and Z component of electric field in equation (12). Using all above relations the Einstein SR coefficient γ is shown to be typical to that of SR.Coefficient γ for particles moving in a field is found by using ordinary relation between velocity, acceleration and potential per unit mass [see equations (1 to 8). fortunately this relation reduces to that of SR in the absence of afield, as equation (9) indicate replacing the velocity v with the average velocity v_m in SR Einstein coefficient in (10), one gets γ in terms ofv_m. Again using the relations between velocity and potential per unit mass, one gets two different expressions for γ depending on the time and time free relation of v and v_0[see equations (19), (26)] . Fortunately the two expressions reduces to that of SR, as shown by equations (20) and (27)

V. CONCLUSION

Lorentz transformation of SR can be found by using Lorentz force expression and Maxwell's equations. The effect of fields on space and time is also derived. It was also found that such transformation reduces to that of SR.

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