Dynamic Interaction of Slip Parameter and Variable Viscosity on MHD Blood Flow in Porous Medium

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ABSTRACT
A mathematical study of pulsatile bio-magnetic blood flow through a channel containing porous medium with variable viscosity is considered. A uniform magneticfield is applied perpendicular to the porous surface. The dimensionless governingcoupled, non-linear partial differential equations are solved by an efficient, accurate and unconditionally stable finite difference scheme ofthe Crank-Nicolson type. The effects of various parameters like Reynolds number,hydro-magnetic parameter, variable viscosity parameter, velocity and thermal slip parameter, Prandtl number and Forchheimer parameter on the velocity and temperaturehavebeen examined with the help of graphs. The present results reported here are likely to have an important bearing on the therapeutic procedure of hyperthermia,particularly in understanding/regulating bloodflow and heat transfer in capillaries.

Keywords: MHD, blood flow, variable viscosity, porous medium.

I. INTRODUCTION

Blood is the primary transport vehicle. It is a suspension of cells in an aqueous solution called “plasma”. There are about $5*10^9$cells in a milliliter of human blood. About 5% of these cells are “platelets” which perform a function related to blood clotting. About 0.2% of the cells are ‘white cells’ which play a role in the resistance of the body to infection. Most of the cells in blood are ‘red cells’ the erythrocytes.The study of blood flow through arteries is of considerable importance in many cardiovascular diseases. The pulsatile flow of blood through an artery has drawn the attention of researchers for a long time due to its great importance in medical sciences. The study of bio-fluids under the presence of magnetic field with dissipation finds its applications in various upcoming fields like innovative drug targeting, surgical operations etc. The presence of electromagnetic fields during such operations can have impacts on the human circulation system. Moreover, apparent viscosity of human blood was found to be significantly influenced by a magnetic field, which may cause a potential human health implication in cases of magnetic fieldexposé for extended times. Haik et al. [1] reported a 30% decrease in blood flow rate when subjected to a high magnetic field of 10 T while Yadav et al. [2] showed a similar reduction in blood flow rate but at a much smaller magnetic field of 0.002 T. Several authors [3-5]have also reported on heat transfer in bio-magneto fluid flows for bio-magnetic convective heat transfer over a stretching surface and bio-magnetic flow and heat transfer in a parallel-plate system.The presence of a porous medium during the flow presents a more physically realizable study. This approach can be modeled in the blood vessels and pulmonary systems due to the presence of fatty deposits and artery blockages. Khaled and Vafai [6]have presented a rigorous review of heat and fluid dynamics applications in porous (biological) media. Generally, a Darcy model is most widely used for modeling the porous conditions but under higher pressure gradients and in highly porous regimes where inertial effects dominate viscous effects, the Darcian model flow is inadequate. The popular approach to simulate drag forces experienced at higher velocities employs the Forchheimer extension to Darcian model and thus known as Darcy-Forchheimer drag model. Various studies using this model have been presented in the context of porous media heat transfer, as described by Pop and Ingham [7]. Significant works include those by Preziosi and Farina [8] who studied mass exchange using an extended Darcy model and Sorek and Sideman [9] who analyzed blood flow in cardiac vessels using a Darcy-Forchheimer model. Recently, Bhargava et.al.[10] used to analyze pulsating magnetohydrodynamic blood flow and species diffusion in a porous medium channel using the Darcy-Forchheimer model. Sharma et al. [11] presented a mathematical model for the hydro-magnetic bio-fluid flow in the porous medium with Joule effect.Blood flow in a large blood vessel has a profound influence on the efficiency of thermal therapy treatment. In pathological situations, thermal radiation therapy is one of the treatments employed by the medical practitioners [12-13]. The procedure involves transmitting heat below the skin surface into tissues and muscles. Deep heat speeds up healing by increasing blood flow to the injury. Electromagnetic heat, such as short waves and microwaves, sends heat up to 2 inches into the tissue and muscles [14]. It works best for injuries in joints, muscles, and tendons. Moreover, hyperthermia treatment has been demonstrated as effective during cancer therapy in recent years. Its objective is to raise the temperature of pathological tissues above cytotoxic temperatures (41–45^{0}C) without overexposing healthy tissues [15].Heat and mass transfer of blood flow considering its pulsatile hydro-magnetic rheological nature under the presence of viscous dissipation, Joule heating and a finite heat source discussed by Sharma et al. [12]. Recently, Sharma and Gaur [16] studied the effect of variable viscosity on chemically reacting magneto-blood flow with heat and mass transfer.The slip condition plays an important role in shear skin, spurt and hysteresis effects. The boundary conditions relevant to flowing fluids are very important in predicting fluid flows in many applications. The fluids that exhibit boundary slip have important technological applications such as in polishing valves of artificial heart and internal cavities [16]. The slip effects on the flow of a non-Newtonian Maxwellian fluid have been investigated by El-Shehawey et al [17]. Sharma and Sharma [18] analysed the effect of variable suction on unsteady free convective flow from a vertical flat plate and heat transfer in slip-flow regime.

The previous studies are based on the constant physical properties of the fluid. However, it is known that the physical properties of the fluid may change significantly with temperature [9]. Since, about 55% of blood is plasma, which contains 92% water.Therefore, to accurately predict the flow and heat transfer rates, it is necessary to take into account the temperature-dependent viscosity of the fluid.Hence, based on the above discussion, the objective of the present work is to study the effects of variable viscosity on MHD heat transfer blood flow in the presence of thermal radiation.

II. MATHEMATICAL FORMULATION

Considered the flow of blood in a straight channel of an artery, by treating blood as a viscous, homogeneous, incompressible fluid. In the Cartesian coordinate system, the x-axis will be assumed along the artery in the direction of flow, and the y-axis will be normal to it. The pulsatile nature of blood will be considered. The fluid subjected to a constant magnetic field acts perpendicular to the artery. Thus, under these assumptions, the problem is governed by the following system of equations:

Linear Momentum Equation

$$\frac{\partial u}{\partial t}+V_0\frac{\partial u}{\partial y}+\frac{1}{\rho}\frac{\partial P}{\partial x}=\frac{\partial}{\partial y}\left(\frac{\mu_a}{\rho}\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2}{\rho}u-\frac{\mu}{\rho k_p}u-bu^2 \quad (1)$$

Energy Equation

$$\frac{\partial T}{\partial t}+V_0\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu_a}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (2)$$

The corresponding boundary conditions on the horizontal plate surfaces are:

$$y=0:\quad u=L_1\frac{\partial u}{\partial y};\quad T=T_1+\delta_T\frac{\partial T}{\partial y};$$
$$y=H:\quad u=0\quad;\quad T=T_2; \quad (3)$$

where $\mu$is assumed to vary as an inverse linear function oftemperature T(see Lai and Kulacki, [19]) as

$$\frac{1}{\mu}=\frac{1}{\mu_a}[1+\gamma(T-T_\infty)]=a(T-T_r),$$

where $a=\gamma/\mu_\infty$and $T_r=T_\infty-1/\gamma$. Both a and $T_\infty$are constants and their values depend on the reference state and the thermal property of the fluid, γ. In general, $a>0$ for liquids and $a<0$ for gases. θ_r is a constant which is defined by

$$\theta_r=\frac{(T_r-T_2)}{(T_1-T_2)}=-\frac{1}{\gamma(T_1-T_2)}.$$

It is worth mentioning that for $\gamma\to 0$, $\mu=\mu_a$,a constant, and $\theta_r\to\infty$. It is also important to note that θ_r is negative forliquids and positive for gases. Here T_2 is free stream temperature.$V_o$is wall transpiration velocity ($V=V_o$,at the lower plate and $V=-V_o$, at the upper plate).

Introducing the following non-dimensional parameters:

$$U=\frac{u}{V_0},\quad X=\frac{x}{H},\quad Y=\frac{y}{H},\quad \tau=\frac{V_0}{H}t,\ P^*=\frac{P}{\rho V_0^2},\quad \theta=\frac{T-T_2}{T_1-T_2},\quad Re=\frac{\rho H V_0}{\mu_a},$$
$$N_m=\frac{\sigma B_0^2 H}{\rho V_0},\quad \lambda=\frac{k_p V_0}{\nu_B H},\quad N_f=Hb,\quad Pr=\frac{\mu}{\rho\alpha},\quad Ec=\frac{V_0^2}{c_p(T_1-T_2)},d_2=\frac{\delta_T}{H},\quad \gamma=\frac{L_1}{H}.$$

Using the expressions of $\theta=\frac{T-T_2}{T_1-T_2}$ and $\theta_r=\frac{(T_r-T_2)}{(T_1-T_2)}$, μ can be written as

$$\mu=\mu_a\left(1-\frac{\theta}{\theta_r}\right)^{-1},\ \text{where}\ \mu_a=-\frac{1}{a\theta_r(T_1-T_2)}. \quad (4)$$

Equations (1), (2)and (4) are reduced to the following non-dimensional form:

Linear Momentum Equation

$$\left(1-\frac{\theta}{\theta_r}\right)^2\left(\frac{\partial U}{\partial \tau}+\frac{\partial U}{\partial Y}+\frac{\partial P}{\partial X}\right)$$
$$=\frac{1}{Re}\left(\left(1-\frac{\theta}{\theta_r}\right)\frac{\partial^2 U}{\partial Y^2}+\frac{1}{\theta_r}\frac{\partial \theta}{\partial Y}\frac{\partial U}{\partial Y}\right)-\frac{1}{\lambda}\left(1-\frac{\theta}{\theta_r}\right)-U\left(1-\frac{\theta}{\theta_r}\right)^2\left(N_m U+\ N_f U^2\right)(5)$$

Energy Equation

$$\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{\partial \theta}{\partial \tau}+\frac{\partial \theta}{\partial Y}\right)=\frac{1}{P_r\,Re}\left(1-\frac{\theta}{\theta_r}\right)\frac{\partial^2 \theta}{\partial Y^2}+\frac{Ec}{Re}\left(\frac{\partial U}{\partial Y}\right)^2 \qquad (6)$$

The transformed boundary conditions now become:

$Y = 0$: $U = \gamma\frac{\partial U}{\partial Y}$; θ = 1+ $d_2\frac{\partial \theta}{\partial Y}$;

$Y = 1$: $U = 0$; θ = 0 ; (7)

As the present problem of fluid flow is pulsatile in nature, therefore the pressure gradient component is decomposed into steady component and the oscillatory component as below:

$$-\frac{\partial P}{\partial X}=\left(\frac{\partial P}{\partial X}\right)_s+\left(\frac{\partial P}{\partial X}\right)_o e^{i\omega t} \qquad (8)$$

III. METHOD OF SOLUTION

In order to solve the unsteady, non-linear coupled equations (5) - (6) under the conditions (7), finite difference scheme has been employed.To obtain the difference equations, the region of the flow is divided into a gird or mesh of lines parallel to Y and t axes. Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. The finite-difference equations at every internal nodal point on a particular n-level constitute a tri-diagonal system of equations. Such systems of equations are solved by using Thomas algorithm as discussed in Carnahan *et al.* [20].

In order to get the accuracy of our results, the computation is carried out for slightly changed values of $\Delta Y$and, Δt. Negligible changed is observedin the values of $U\, and \theta$and also after each cycle of iteration the convergence checking is performed i.e. $|f^{n+1}-f^n|<10^{-7}$is satisfied at all points. Thus, due to computational cost and accuracy considerations the above mesh size was as the optimal.

IV. RESULTS AND DISCUSSION

MHD bio-fluid flow with variable viscosity has been carried out in preceding sections. In order to get physical insight into the problem, the numerical calculations for the distribution of the velocity and temperature for various dimensionless parameters.

Fig. 1 represents the velocity profile U for different values of Reynolds number (R_e) at $t = 0.5$. As Reynolds number involves transpiration velocity, hence increase in R_e results in increase of velocity (U) and peaks attain at the mid of the channel.

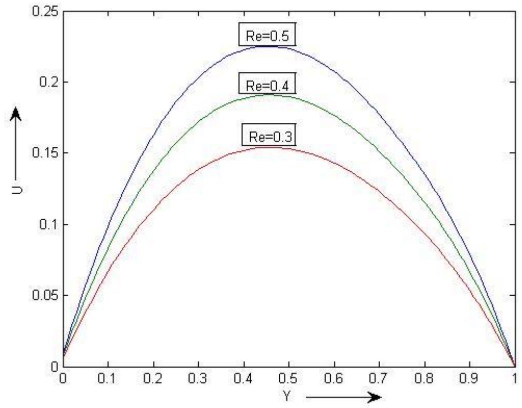

Fig.1. Velocity profile for different values of variable of transpiration Reynolds numbers (R_e) viscosity parameter

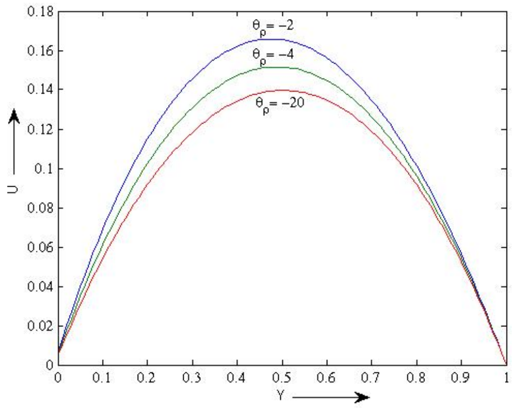

Fig.2 Velocity profile for different values

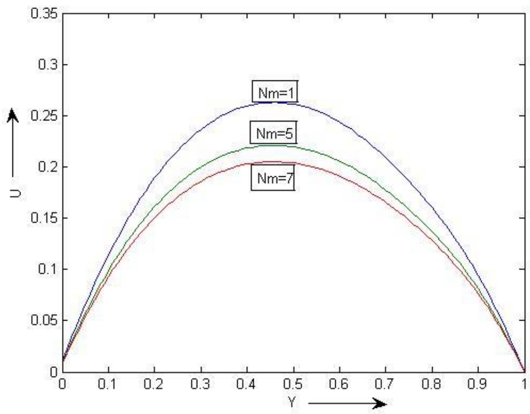

Fig. 3. Velocity profile for different values of hydromagnetic parameter values (Nm)

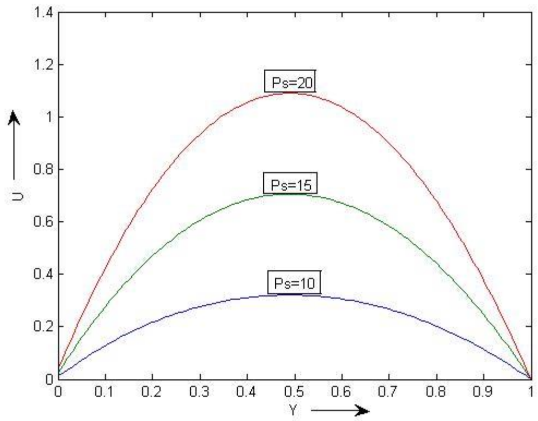

Fig. 4. Velocity profile for different values of steady pressure gradient values (P_s)

The effect of variable viscosity parameter on velocity profile is shown in Fig. 2. It is observed that an increase in viscosity parameter results a decrease in velocity distribution. The effect of hydro-magnetic parameter (N_m) is depicted in Fig. 3.It is observed that the velocity decreases with increasing N_m. This is due to the retarding forces (Lorentz forces) generated by the magnetic field as bio-fluid's electrical properties have been already confirmed.Fig. 4 shows the influence of steady component of pressure (P_s) on the velocity profiles. As P_s increases from 10 to 15, there is a strong rise in velocity as high pressure conditions force fluid to flow at higher velocities.

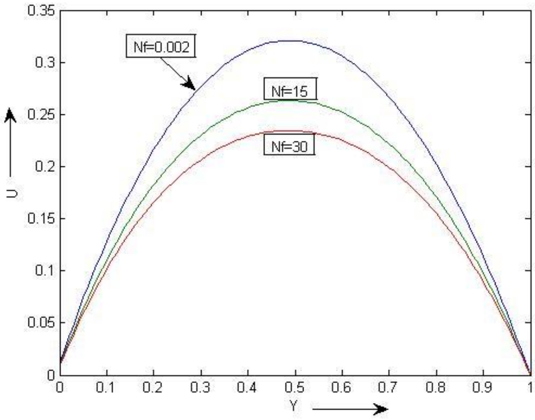

Fig .5 Velocity profile for different values of Forchheimer parameter

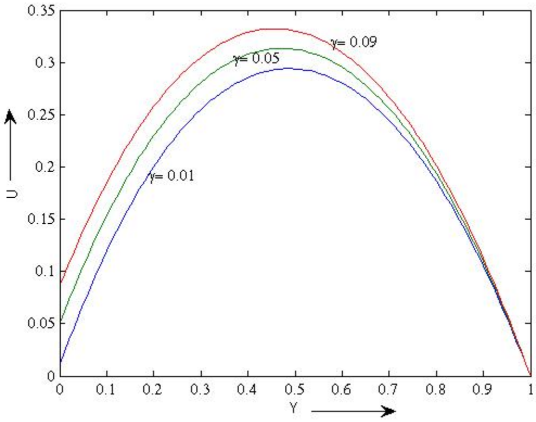

Fig 6. Velocity profile for different values of slip parameter

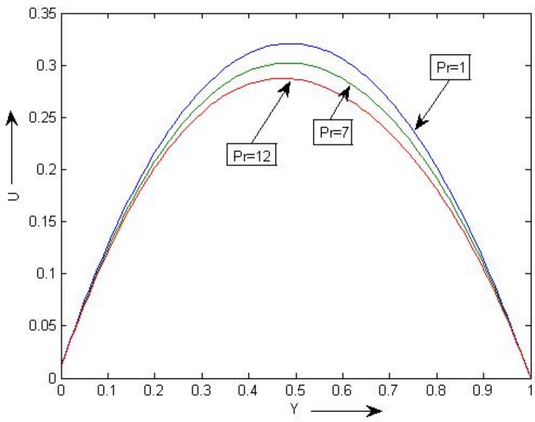

Fig .7 Velocity profile for different values of Prandtl number

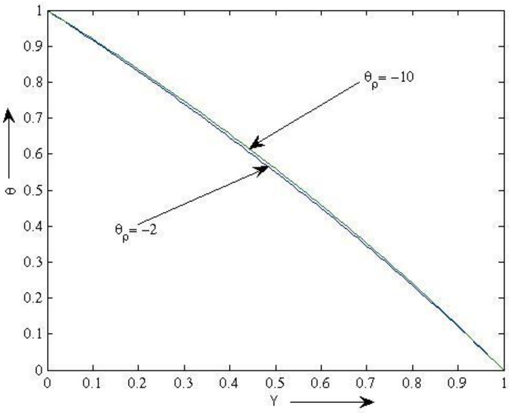

Fig 8. Temperature profile for different values of variable viscosity parameter

The effects of Forchheimer drag parameter (N_f) on velocity profile are shown in fig. 5. As N_f increase, the velocity profiles reduced because higher N_f means higher drag values and thus resulting in lower velocities in the channel.The effect of velocity slip parameter is shown in Fig. 6. By increasing the slip at the wall the velocity increases which is in accordance to the existing results in literature. It is also observed that the velocity is smaller near beginning and ending of the wave but becomes larger in the central region. The velocity profile for different values of Prandtl number is shown in Fig. 7. It is noted that velocity decreases as Pr increases.

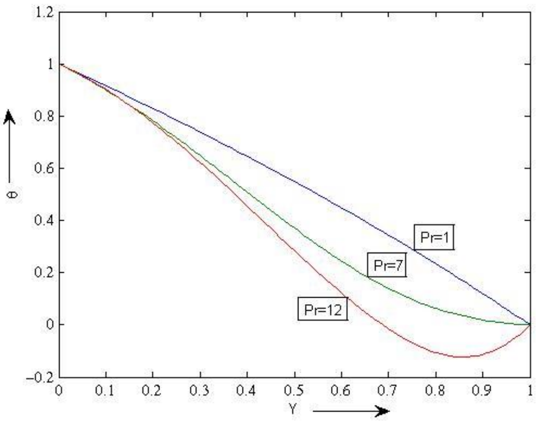

Fig. 9. Temperature profile for different values of Prandtl number (P_r)

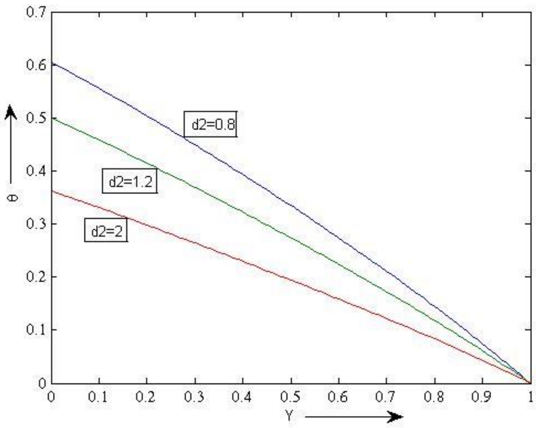

Fig. 10. Temperature profile for different values of thermal slip parameter

The effect of variable viscosity parameter is shown in Fig. 8. It is observed that an increase in viscosity parameter results a decrease in temperature distribution. The effect of Prandtl number (P_r) on temperature distribution is shown in Fig. 9. P_r represents the ratio of momentum diffusivity to thermal diffusivity. Larger P_r fluids (P_r>1) will diffuse momentum faster than heat. The numerical results show that the temperature decreases with increase in the Prandtl number.The effect of thermal slip parameter (d_2) on temperature distribution is shown in Fig. 10. It is noted that the temperature profile decreases with increasing thermal slip parameter.

Nomenclature
ρ=density of the fluid,
τ=dimensional time,
σ=electrical conductivity of the bio-fluid,
γ=velocity slip parameter
λ=Darcian (permeability) parameter,
θ=dimensionless temperature,
μ_B=Newtonian dynamic viscosity,
ω=dimensionless angular frequency,
c_p=specific heat capacity of the bio-fluid,
d_2=thermal slip parameter
$\partial P/\partial X$=longitudinal pressure gradient
$P*$=transformed hydrodynamic pressure (* dropped for convenience in analysis),
R_e= Reynolds number,
b=Forchheimer coefficient related to the porous medium geometry,
u=x-direction (longitudinal velocity),
k_p=hydraulic conductivity (permeability) of the porous material,
A=thermal diffusivity,
B_0=transverse magnetic field strength,
E_c=Eckert Number
Nf=Forchheimer (quadratic porous drag) parameter/ number,
N_m=hydro-magnetic parameter,
P=hydrodynamic pressure,
P_s=steady component of pressure gradient
P_0 = oscillatory pressure component.
P_r= Prandtl number,
t=dimensionless time,
T=bio-fluid temperature,
V_o=wall transpiration velocity ($V = V_o$at the lower plate and $V = -V_o$at the upper plate),
U=transformed velocity component in the X-direction,
X= dimensionless coordinate parallel to the bio-fluid flow
Y= dimensionless coordinate transverse to the bio-fluid flow

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