

Modeling MHD Nanofluid Flow over a Moving Surface Influenced by Thermal Radiation Using HAM

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ABSTRACT

In this paper the effect of Brownian motion and Thermophoresis on boundary layer flow of MHD nanofluid flow over a moving surface with the influence of thermal radiation are studied. The governing partial differential equations are transformed into ordinary differential equations, by using a similarity transformation. The resulting ordinary differential equations are solved by homotopy analysis method. We observed physical insight and interesting aspects of the problem in presence of thermal radiation. The effect of different non-dimensional parameters are studied namely Prandtl number Pr , Brownian motion parameter N_b , Thermophoresis parameter N_t , Lewis number Le , Hartman number Ha and Radiation parameter R are depicted graphically for velocity, temperature and nanoparticle concentration profile.

Keywords: Thermophoresis, Brownian motion, Nanofluid, Thermal radiation, HAM, MHD.

1. INTRODUCTION

Heat can be transferred from one place to another by conduction in solids, convection in liquids or gases and radiation for which no media is required. Thermal radiation is the fundamental mechanisms of heat transfer. Heat generation and heat transfer are very important characteristic phenomenon involved in many engineering and industrial process. Olanerwaju et al [16] gave the concept of boundary layer flow of nanofluids over a moving surface in presence of thermal radiation. In some industries enormous heat will be generated and transferred through thermal radiation which may lead to many damages in process of mechanics. To reduce this thermal radiation one has to use some coolant in machineries.

Nanofluids are made of nanoparticles (<100nm) suspended in a base fluids such as water, oil and ethylene glycol. Recently nanotechnology has invented nanofluids discovered by Choi's team [4]. There exist a nano layer that acts as a thermal bridge between solid nanoparticles and base fluid which plays significant role in heat transfer. It can be used as coolant in industries resulting in energy saving. Ahmadsreza [22] observed that due to their higher thermal conductivity nanofluids are used as liquid cooling. Azwadi Che Sidik [27] noted the applications of nanofluids and various minichannel configurations for heat transfer improvement. Nanofluids have heat transfer applications as a microelectronic fuel cells, electronic cooling, domestic refrigerator chillers, solar water heating chillers, heat pipes, lubrication, oil recovery, detergency and processes of soil remediation,. Husam Abdulrasool Hasan et al. [28] discussed the heat transfer enhancement using nanofluids for Cooling.

One of the recent Bio-medical applications used to kill cancer cells is Nano-cryosurgery which is also called cryotherapy or cryodestruction or cryoablation. Jing Liu and Zhong-Shan Deng [12] highlighted the advances and challenges in Nano-cryosurgery. De-Rui Di et al. [18] explained experimentally that cryosurgery uses freezing to destroy undesirable tissues by using MgO nanoparticles were adopted as a biodegradable nontoxic agent. Chandran and Senthil Kumar [20] explained that in surgical procedure by using cryosurgery device called cryoprobe in cancer treatment to destroy undesirable tumor cells. By introduction of nanoparticles into target tissue area, it improves image contrast and gives the guidance for Cryosurgery.

Crane [2] investigated the concept of fluid flow over a stretching sheet. The flow of various non-Newtonian fluids over stretching sheet was analyzed by Liao [9]. Concept of boundary layer flow over a moving surface was presented by Sakiadis [1].

In many physical and engineering applications such as plasma physics, packed-bed catalytic reactors, thermal insulation, geothermal reservoirs etc, the applications of MHD boundary layer flow of heat and mass transfer are

found. Bachok et al. [13] studied boundary layer flow of nanofluid over a moving surface. Anuar Ishak [16] studied the effect of radiation on MHD boundary layer flow of a viscous fluid over an exponentially stretching sheet. Shadloo et al. [15] applied HAM to obtain the analytical solutions in the study of a two-dimensional steady convective flow of a micro and engineering fluid flows such as polymeric fluids, liquid crystals, paints colloidal fluids. Eshetu Haile and Shankar [23, 26] studied boundary layer flow of nanofluids through a porous medium subjected to a magnetic field, thermal radiation, viscous dissipation and chemical reaction effects. They observed that Brownian effect and thermophoresis effect improves the heat transfer rate. Nick Kaiser [8] explained that the Stefan-Boltzmann law gives the energy of thermal radiation emitted per unit time by a black body of a surface area which is directly proportional to fourth power of the temperature.

II. BASIC EQUATIONS

Consider the steady MHD boundary layer flow of a nanofluid in a uniform free stream in presence of thermal radiation whose velocity is assumed as U , towards a moving surface velocity is given by $U_w = \lambda U$ where λ is the velocity parameter. $\lambda > 0$ corresponds to the downstream movement of the plate from the origin and $\lambda < 0$ corresponds to the upstream movement of the plate. The flow being confined at $y \geq 0$. it is assumed that y coordinate is measured normal to the moving surface and uniform magnetic field is applied in 'y' direction. Let T_w and C_w are the temperature of fluid and nanoparticle fraction at wall. For free stream values are taken to be T_∞ and C_∞ . Tiwari and Das [11] proposed the nanofluid model with Boussinesq and boundary layer approximation. The geometry of flow is shown in figure1. Continuity, momentum and energy equation for the steady MHD boundary layer flow of a nanofluid past a moving semi infinite flat plate in a uniform stream in presence of thermal radiation are given by

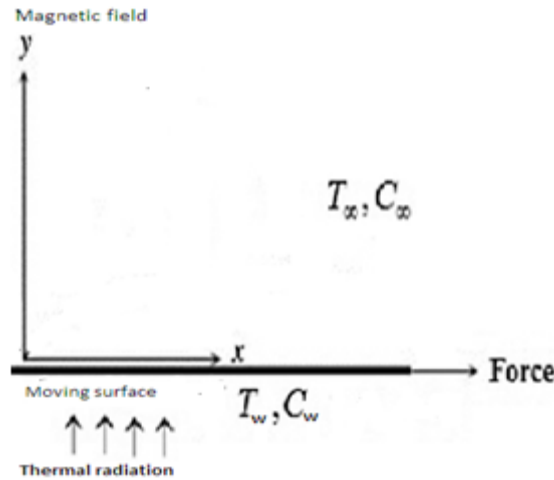


Figure1: Physical model and coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_f} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{\alpha}{k} \frac{\partial q_r}{\partial y} + \tau [D_B \left(\frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial T}{\partial y} \right)^2], \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

where u is the velocity component along the x -axis, v is the velocity component along the y -axis, ν is the kinematic viscosity coefficient, k is the thermal conductivity, q_r radiative heat flux, B_0 is the uniform magnetic field strength of the base fluid, σ is the electrical conductivity of the base fluid, τ is the ratio of the nanoparticle heat capacity and the base fluid heat capacity, q_w is the heat flux, q_m Mass flux, $\alpha = k/(\rho c)_f$ is the thermal diffusivity of the fluid. D_B the Brownian diffusion coefficient, D_T the Thermophoresis diffusion coefficient,

The associated boundary conditions are taken as
 $v = 0, u = \lambda U, T = T_w, C = C_w$ at $y = 0$

(5)

$U \rightarrow U, T \rightarrow T_\infty, C \rightarrow C_\infty$ as $y \rightarrow \infty$.

(6)

By using Rosseland diffusion approximation [5, 6, 8] the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma^*}{3K_s} \frac{\partial T^4}{\partial y}, \quad (7)$$

Where σ^* and K_s are the Stefan-Boltzman constant and the Rosseland mean absorption coefficient respectively. We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \quad (8)$$

Using (7) and (8) in the last term of equation (3), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*T_\infty^3}{3K_s} \frac{\partial^2 T}{\partial y^2}. \quad (9)$$

To reduce the governing equations into a system of ordinary differential equations, we introduce the following Similarity transformations.

$$\Psi = (2U\nu x)^{\frac{1}{2}} \phi(\eta), \quad \psi(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (10)$$

$$\xi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \eta = (U/2\nu x)^{1/2} y. \quad (11)$$

where $\phi(\eta)$, $\psi(\eta)$ and $\xi(\eta)$ are the dimensionless stream function, temperature, and nanoparticle concentration respectively and η is the similarity variable.

The stream function Ψ is defined as

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x}. \quad (12)$$

Using similarity transformation and associated boundary conditions, the continuity equation is identically satisfied. Momentum equation and energy equation reduces to ODE.

The governing coupled non linear equations for this problem are

$$\phi''' + \phi\phi'' - \text{Ha}(\phi') = 0, \quad (13)$$

$$\frac{(3+4R)}{\text{Pr}} \psi'' + \phi\psi' + N_b \xi'\psi' + N_t \psi'^2 = 0, \quad (14)$$

$$\xi'' + \text{Le} \phi\xi' + \frac{N_t}{N_b} \psi'' = 0, \quad (15)$$

$$\phi(0) = 0, \phi'(0) = \lambda, \phi' \rightarrow 1 \text{ as } \eta \rightarrow \infty, \quad (16)$$

$$\psi(0) = 1, \psi \rightarrow 0 \text{ as } \eta \rightarrow \infty, \quad (17)$$

$$\xi(0) = 1, \xi \rightarrow 0 \text{ as } \eta \rightarrow \infty, \quad (18)$$

where

$$R = \frac{4\alpha\delta T_\infty^3}{kk}, \text{Pr} = \frac{\nu}{\alpha}, \text{Le} = \frac{\nu}{D_B}, \quad (19)$$

$$\text{Ha} = \frac{2xB_0^2}{U\rho_f}, N_b = \frac{(\rho c)_p D_B (\phi_w - \phi_\infty)}{\nu(\rho c)_f}, N_t = \frac{(\rho c)_p D_T (T_w - T_\infty)}{\nu(\rho c)_f T_\infty} \quad (20)$$

N_b is the Brownian motion parameter, N_t is the Thermophoresis parameter, τ_w is the Shear stress, R the Radiation parameter, Pr the Prandtl number, Le Lewis number, Ha the Hartman number

Limiting case: Stanford Shately and Jagadish Prakash [24] in which spectral relaxation method is applied directly to ODE. Eshetu Haile and Shankar B [23, 26] in which shooting technique along with the fourth order Runge-Kutta

integration scheme method is applied directly to non linear ODE. But here we applied semi analytical method HAM to ODE directly, which exactly matches with earlier result

III. METHODOLOGY

Homotopy Analysis Method for nonlinear boundary value problems are discussed in [10, 21].

The region of convergence and the rate of convergence can be adjusted by varying auxiliary linear operator L , auxiliary parameter h and initial solution. Liao [10] determined the optimal value of h involves plotting the h -curves of the solution. We can expect to see the horizontal over the range for which the solution converges. In figure 2 we have plotted $\phi''(0)$ against auxiliary parameter h . similarly we can obtain for temperature profile and nanoparticle profile.

The governing coupled non linear equations for this problem are written as

$$N[\phi(\eta)] = \phi''' + \phi\phi'' - \text{Ha}(\phi'), \quad (21)$$

$$N[\psi(\eta)] = \frac{(3+4R)}{Pr}\psi'' + \phi\psi' + N_b \xi'\psi' + N_t\psi'^2, \quad (22)$$

$$N[\xi(\eta)] = \xi'' + \text{Le}\phi\xi' + \frac{N_t}{N_b}\psi''. \quad (23)$$

For HAM solution an auxiliary linear operator for the equation (21), (22), (23) respectively as

$$L_\phi = \frac{\partial^3}{\partial \eta^3} + \frac{\partial^2}{\partial \eta^2}, L_\psi = \frac{\partial^2}{\partial \eta^2} + \frac{\partial}{\partial \eta}, L_\xi = \frac{\partial^2}{\partial \eta^2} + \frac{\partial}{\partial \eta}. \quad (24)$$

Consider $L_\phi[\phi] = 0$, $L_\psi[\psi] = 0$, $L_\xi[\xi] = 0$ and using boundary conditions (16), (17), (18) for ϕ , ψ , ξ we get the initial approximations are $\phi_0(\eta)$, $\Psi_0(\eta)$, $\xi_0(\eta)$ as

$$\phi_0(\eta) = f_0(\eta) = (\lambda - 1) + \eta + (1 - \lambda)e^{-\eta}, \quad (25)$$

$$\Psi_0(\eta) = \theta_0(\eta) = e^{-\eta}, \quad (26)$$

$$\xi_0(\eta) = \varphi_0(\eta) = e^{-\eta}. \quad (27)$$

As p varies from 0 to 1, the solution $\phi_0(\eta)$, $\Psi_0(\eta)$, $\xi_0(\eta)$ varies from the initial guess to the exact solution $\phi(\eta)$, $\psi(\eta)$, $\xi(\eta)$.

Homotopy equations for (27), (28), (29) are constructed as below

$$(1 - p)L[A(\eta, p) - \phi_0(\eta)] = hp \left\{ \frac{\partial^3 A}{\partial \eta^3} + A \frac{\partial^2 A}{\partial \eta^2} - \text{Ha} \left(\frac{\partial A}{\partial \eta} \right) \right\}, \quad (28)$$

$$(1 - p)L[B(\eta, p) - \Psi_0(\eta)] = hp \left\{ \left(\frac{3+4R}{Pr} \right) \frac{\partial^2 B}{\partial \eta^2} + A \frac{\partial B}{\partial \eta} + N_b \frac{\partial C}{\partial \eta} \frac{\partial B}{\partial \eta} + N_t \left(\frac{\partial B}{\partial \eta} \right)^2 \right\}, \quad (29)$$

$$(1 - p)L[D(\eta, p) - \xi_0(\eta)] = hp \left\{ \frac{\partial^2 D}{\partial \eta^2} + \text{Le}A \frac{\partial D}{\partial \eta} + \frac{N_t}{N_b} \frac{\partial^2 B}{\partial \eta^2} \right\}. \quad (30)$$

When $p=0$ and $p=1$ we have

$$A(\eta, 0) = \phi_0(\eta) \quad A(\eta, 1) = \phi(\eta), \quad (31)$$

$$B(\eta, 0) = \Psi_0(\eta) \quad B(\eta, 1) = \psi(\eta), \quad (32)$$

$$D(\eta, 0) = \xi_0(\eta) \quad D(\eta, 1) = \xi(\eta). \quad (33)$$

Obviously conditions are,

$$A(0, p) = 0, A_\eta(0, p) = \lambda, A_\eta(\infty, p) = 1, \quad (34)$$

$$B(0, p) = 1, B(\infty, p) = 0, \quad (35)$$

$$D(0, p) = 1, D(\infty, p) = 0. \quad (36)$$

Applying Maclaurin's series expansion to $A(\eta, p)$, $B(\eta, p)$ and $D(\eta, p)$ and using initial approximation (25)-(27) we get

$$A(\eta, p) = f_0(\eta) + \sum_{k=1}^{\infty} f_k(\eta) p^k, \quad (37)$$

$$B(\eta, p) = \theta_0(\eta) + \sum_{k=1}^{\infty} \theta_k(\eta) p^k, \quad (38)$$

$$D(\eta, p) = \varphi_0(\eta) + \sum_{k=1}^{\infty} \varphi_k(\eta) p^k. \quad (39)$$

The convergence region of the above series depends upon the auxiliary linear operator L , and the non-zero auxiliary parameter h which are to be selected such that solution converges at $p = 1$.

$$\phi(\eta) = f_0(\eta) + \sum_{k=1}^{\infty} f_m(\eta), \quad (40)$$

$$\psi(\eta) = \theta_0(\eta) + \sum_{k=1}^{\infty} \theta_m(\eta), \quad (41)$$

$$\xi(\eta) = \varphi_0(\eta) + \sum_{k=1}^{\infty} \varphi_m(\eta). \quad (42)$$

Differentiating equation (28), (29) and (30) m times about the embedding parameter p , using Leibnitz theorem, setting $p = 0$ and dividing by $m!$ We get

$$L[f_m - \chi_m f_{m-1}] = h R_m(\eta), \quad (43)$$

$$L[\theta_m - \chi_m \theta_{m-1}] = h S_m(\eta), \quad (44)$$

$$L[\varphi_m - \chi_m \varphi_{m-1}] = h T_m(\eta). \quad (45)$$

where $\chi_m = \begin{cases} 0 & \text{when } m \leq 1 \\ 1 & \text{when } m > 1 \end{cases}$ and

$$R_m(\eta) = f_{m-1}'''(\eta) + \sum_{k=0}^{m-1} f_{m-1-k}(\eta) f_k''(\eta) - H a f_{m-1}'(\eta), \quad (46)$$

$$S_m(\eta) = \left(\frac{3+4R}{Pr} \right) \theta_{m-1}''(\eta) + \sum_{k=0}^{m-1} f_{m-1-k}(\eta) \theta_k'(\eta) + N_b \sum_{k=0}^{m-1} \varphi_{m-1-k}'(\eta) \theta_k'(\eta) - N_t \sum_{k=0}^{m-1} \theta_{m-1-k}'(\eta) \theta_k'(\eta), \quad (47)$$

$$T_m(\eta) = \varphi_{m-1}''(\eta) + L e \sum_{k=0}^{m-1} f_{m-1-k}(\eta) \xi_k'(\eta) + \frac{N_t}{N_b} \theta_{m-1}''(\eta), \quad (48)$$

with boundary conditions

$$f_m(0) = 0, f_m'(0) = 0, f_m'(\infty) = 0, \quad (49)$$

$$\theta_m(0) = 0, \theta_m(\infty) = 0, \quad (50)$$

$$\varphi_m(0) = 0, \varphi_m(\infty) = 0. \quad (51)$$

We solve these non-linear equations given by (43), (44) and (45) for f_m, θ_m, φ_m by MATHEMATICA. For the given equations we get the solution for the different characteristic parameters we analyze the solutions through graphs.

IV. RESULT AND DISCUSSION

The solutions for the (46)-(48) system of equation with corresponding boundary conditions are obtained by homotopy analysis method. With the help of Mathematica equations are solved and discussed through graphs. The convergence of the equation depends on varying auxiliary linear operator L , auxiliary parameter h and initial solution. The velocity profile $\Phi(\eta)$ of nanofluid decreases by increase in the value of Hartman number Ha as depicted in fig3. In fig 4 we observed that by increase in the value of velocity parameter λ the fluid velocity increases. Fig5 gives the effect of Hartman number Ha on the temperature profile $\Psi(\eta)$, which increases with the increase in the value of magnetic field parameter Ha , here the applied magnetic field tends to heat the fluid which reduces the heat transfer from the wall. Whereas The effect of magnetic field parameter Ha lowers the value of nanoparticle volume fraction as in fig 9. In fig 6 Temperature profile $\Psi(\eta)$ increases by increase in the value of radiation parameter R , the thermal radiation enhances the thermal diffusion. In fig 7 and fig 8 we notice, the influence of thermophoresis parameter N_t and of Brownian motion parameter N_b enhance the temperature $\Psi(\eta)$ of the nanofluid. Whereas increases in the value of Lewis number Le results in thinning of the boundary layer which can be shown in fig 10. In fig 11 increase in the value of thermophoresis parameter N_t increases the nanoparticle volume fraction profile.

V. CONCLUSION

Present study we have applied HAM where the solution exactly matching with numerical result obtained by previous [36] author in which spectral relaxation method is applied directly to ODE. Here we observed that the semi analytical method HAM works well for non-linear differential equations

VI. GRAPHS

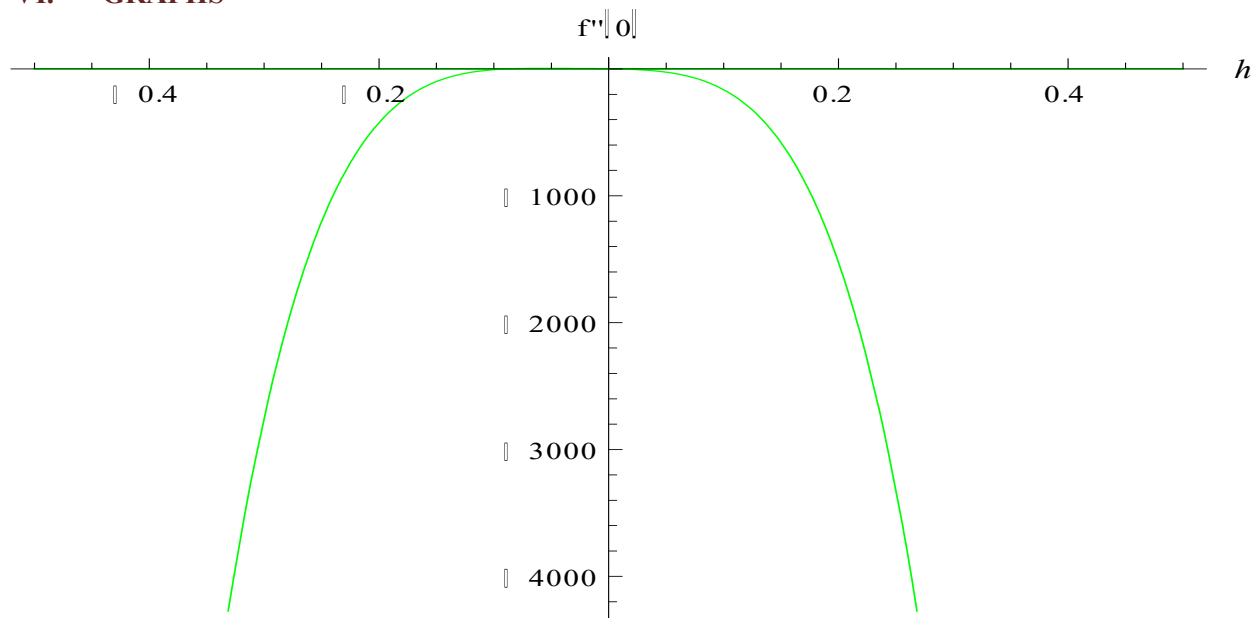


Figure 2 h-curve for velocity profile

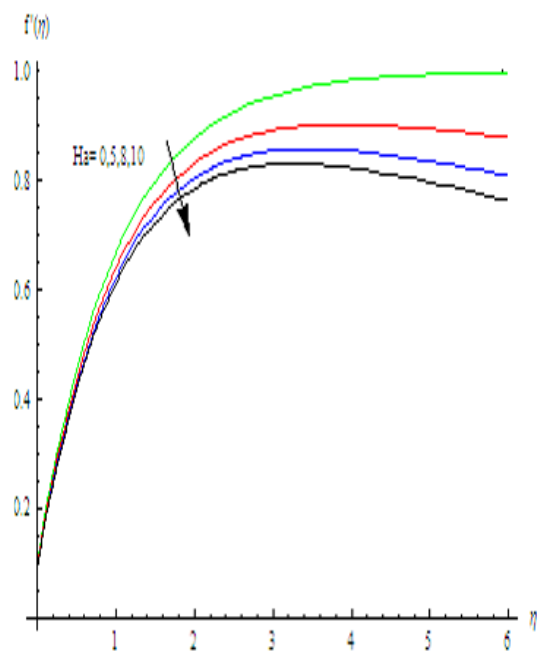


Fig3. Velocity profiles for different values of Ha

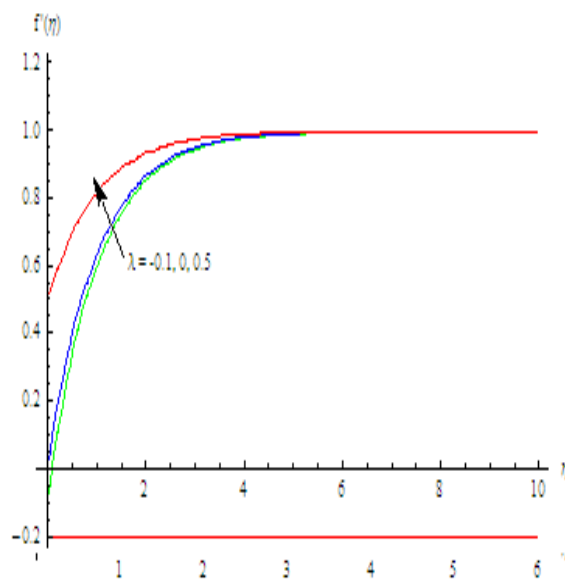


Fig4. Velocity profiles for different values of λ

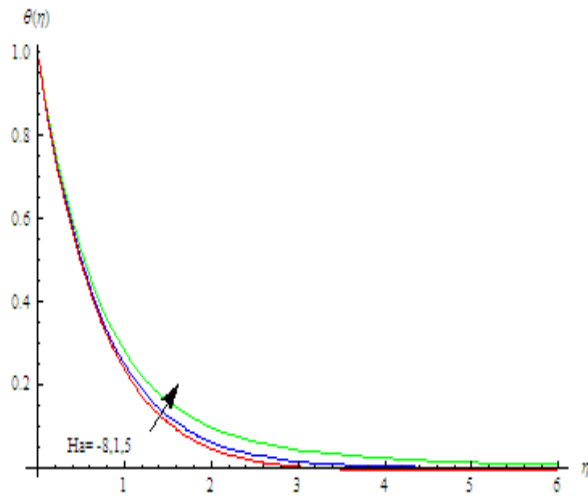


Fig5. Temperature distribution for different values of Ha

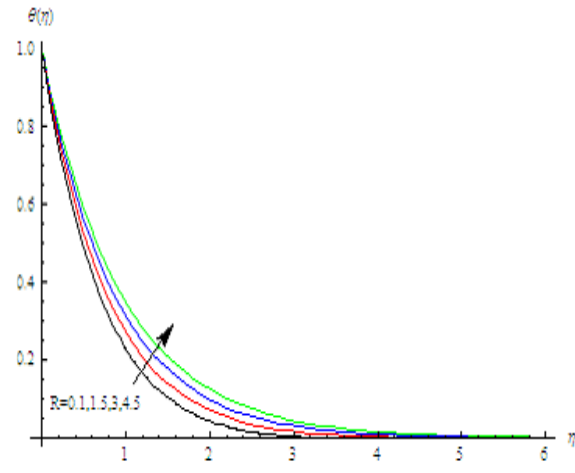


Fig6 Temperature distribution for different values of R

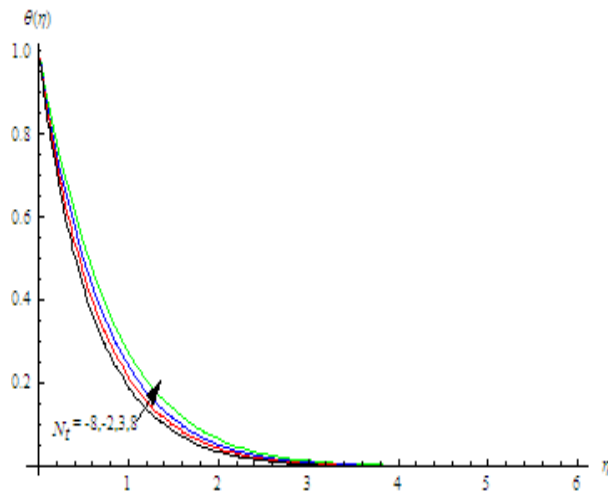


Fig7. Temperature distribution for different values of N_t

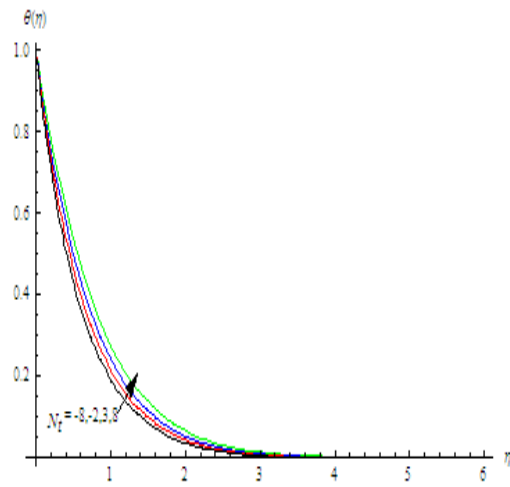


Fig8 Temperature distribution for different values of N_b

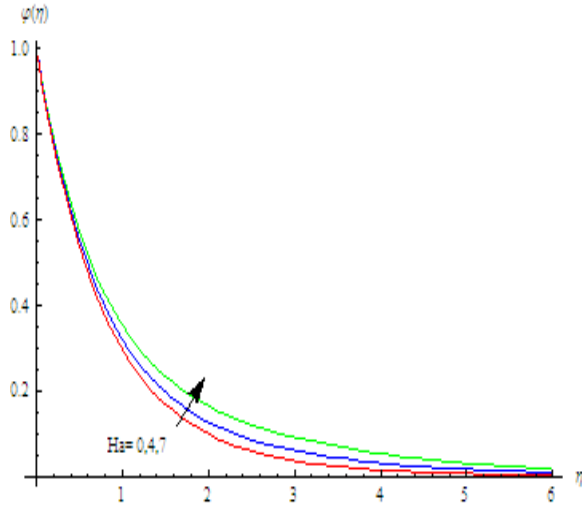


Fig9. Nano particle fraction for different values of Ha

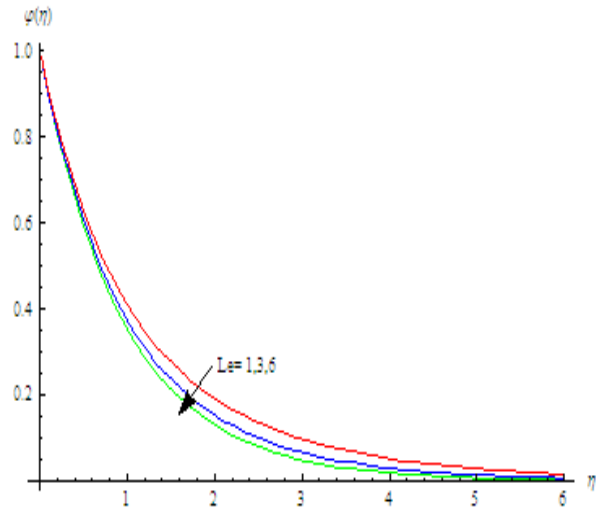


Fig10. . Nano particle fraction for different values of Le

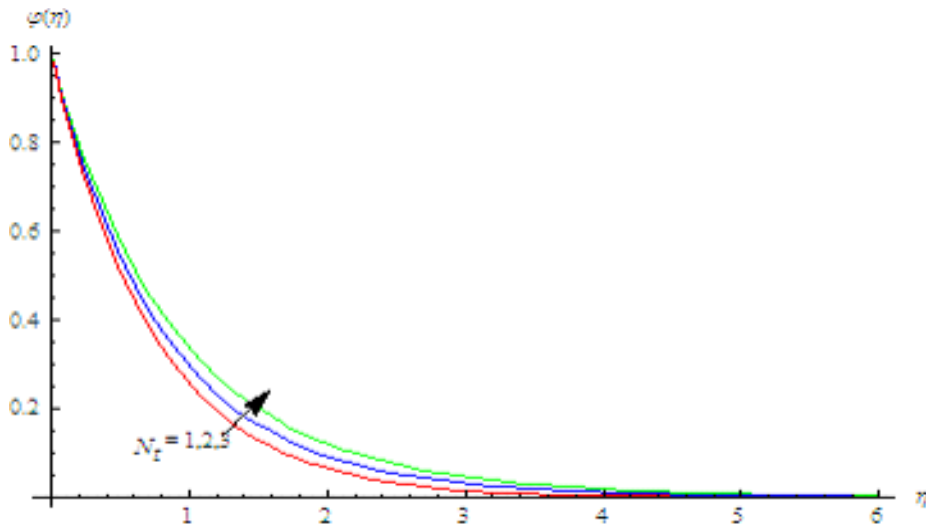


Fig11. Nano particle fraction for different values of N_t

Nomenclature:

u Velocity components along the x-axis	v Velocity components along the y-axis,
ν Kinematic viscosity coefficient,	k Thermal conductivity,
D_B Brownian diffusion coefficient,	D_T Thermophoresis diffusion coefficient,
B_0 Uniform magnetic field strength of the base fluid,	σ Electrical conductivity of the base fluid,
$\alpha = k/(\rho c)_f$ is the Thermal diffusivity of the fluid,	Pr Prandtl number
N_t Thermophoresis parameter	Ha Hartman number
τ_w Shear stress	R Radiation parameter
Le Lewis number	q_w Heat flux
q_m Mass flux	q_r Radiative heat flux
N_b Brownian motion parameter	
τ Ratio of the nanoparticle heat capacity and the base fluid heat capacity.	

REFERENCE

1. Sakiadis BC "Boundary-layer behavior on continuous solid surface: I Boundary-layer equations for two-dimensional and axisymmetric flow", J Am Inst Chem Eng vol7, (1961); pp 26–28.

2. Crane L, Flow past a stretching plate. *Z Angew Math Phys* vol21 (1970); pp. 645–647.
3. S. J. Liao “The proposed homotopy analysis techniques for the solution of nonlinear problems”, Ph.D. Dissertation, Shanghai Jiao Tong University, Shanghai, China
4. Choi SUS “Enhancing thermal conductivity of fluids with nanoparticles”, *International Mechanical Engineering Congress and Exposition*, San Francisco, USA, ASME, FED 231/MD, vol66, (1995); pp. 99–105.
5. Hossain. M. A. and Takhar, H. S. Radiation effect on mixed convection along a vertical plate with uniform surface temperature. *Heat Mass Transf.* 31, (1996); pp. 243–248.
6. Raptis, A. “Flow of a micropolar fluid past a continuously moving plate by the presence of radiation”, *Int. J. Heat Mass Transf.* 41, (1998); pp.2865–2866.
7. Rami, Y. Jumah, et al. “Darcy-Forchheimer mixed convection heat and mass transfer in fluid saturated porous media”, *Int. J. of Num. Meth. for Heat and Fluid Flow*, vol. 11, issue 6. (2001); pp600–618.
8. Nick Kaiser “Elements of Astrophysics” Publisher: University of Hawaii (2002)
9. S. J. Liao. On the analytic solution of magneto hydro dynamic flows of non-Newtonian fluids over a stretching sheet”, *J of fluid mechanics*, vol488, (2003); pp. 189–212.
10. S. J. Liao “Beyond Perturbation: Introduction to the Homotopy Analysis Method”, vol2 of CRC Series: *Modern Mechanics and Mathematics*, Chapman and Hall/CRC, Boca Raton, Fla, USA (2004)
11. Tiwari. R. K. and Das. M. K. “Heat transfer augmentation in a two sided lid-driven differentially heated square cavity utilizing nanofluids”, *Int. J. Heat mass transfer*, vol. 50, (2007), pp. 2002–2018.
12. Jing Liu and Zhong-Shan Deng, “Nanocryo-surgery: Advances and challenges”, *Journal of Nanosciences and nanotechnology*, vol. 9, 1–22.
13. Bachok. N. “Boundary layer flow of nanofluids over a moving surface in a flowing fluid”, *Int. J. of Thermal Sci.*, vol.49, issue9, (2010); pp.1663–1668.
14. Achala.L.N and Sathyanarayana.S.B, “Fluid over nonlinearly stretching sheet with magnetic field by homotopy analysis method”, *Jl of Applied Mathematics and fluid mechanics* , vol3, issue1, (2011); pp.15–22,
15. Shadello. M. S. et al. “Series solution for heat transfer of continuous stretching sheet immersed in a micro polar fluid in the existence of radiation”, *international journal of numerical methods for heat and fluid flow*”, vol23, issue2, (2011); pp.289–304.
16. Anuar Ishak “MHD boundary layer flow due to an exponentially stretching sheet with radiation effect”, *Sains Malaysia* vol40 issue4, (2011); pp.391–395.
17. Olanrewaju. P. O. et al “Boundary layer flow of nanofluids over a moving surface in a flowing fluid in the presence of radiation”, *International Journal of Applied Science and Technology*, vol2, issue1, (2012); pp.284–295
18. De-Rui Di et al. “A new nano-cryosurgical modality for tumor treatment using biodegradable MgO nanoparticles”, *Nanomedicine: Nanotechnology, Biology and Medicine*, 8, (2012); pp. 1233–1241
19. V. G. Gupta and Sumit Gupta, Application of homotopy analysis method for solving nonlinear Cauchy problem, *surveys in mathematics and its applications* , vol7, (2012), pp. 106–116.
20. Chandran and Senthil kumar “Cryoprobe a tool for cryo(nano)surgery and cryonics, *International journal of Advancement in Research and Technology*, vol2, issue4, (2013); pp. 305–308
21. Achala. L. N and Sathyanarayana. S. B, “Nonlinear boundary value problems by homotopy analysis method”, *Jl Applied Mathematics*, vol5, (2013), pp.27–46.
22. Ahmadreza, A. B. “Application of nanofluid for heat transfer enhancement”, PID: 2739168, EEE-5425.
23. Eshetu Haile and Shankar. B. “Heat and mass transfer through a porous media of MHD flow of nanofluids with thermal radiation, Viscous Dissipation and chemical reaction effects”, *American chemical science journal* vol4, issue6, (2014); pp. 828–846.
24. Stanford Shately and Jagadish Prakash “A new numerical approach for MHD laminar boundary layer flow and heat transfer of nano fluids over a moving surface in presence of thermal radiation”, *Springer open journal* . (2014)
25. Liarn morrow and Matthew Simpson, *An investigation of the Homotopy Analysis Method for solving non-linear differential equations*, AMSI, (2014).
26. Eshetu Haile and Shanker. B. “Boundary-Layer Flow of Nanofluids over a moving surface in the presence of Thermal Radiation, Viscous Dissipation and Chemical Reaction”, *Application and applied Mathematics: An International journal (AAM)*, vol10, issue 2, (2015); pp.952–969.

27. Nura Mu'az Muhammad and Nor Azwadi Che Sidik. "Applications of Nanofluids and Various Minichannel Configurations for Heat Transfer Improvement: A Review of Numerical Study, *journal of Advanced Research in Fluid Mechanics and Thermal Sciences* ", vol. 46, Issue 1), (2018); pp.49-61.
28. Husam Abdulrasool Hasan, Zainab Alquziweeni, and Kamaruzzaman Sopian. " Heat Transfer Enhancement Using Nanofluids for Cooling a Central Processing Unit (CPU) System", *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* , vol. 51, issue 2, (2018); pp.145-157.